**Computer Methods of Analysis of Offshore Structures**
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**Module – 01**
**Lecture – 25**
**Analysis of Space Frame Structures**

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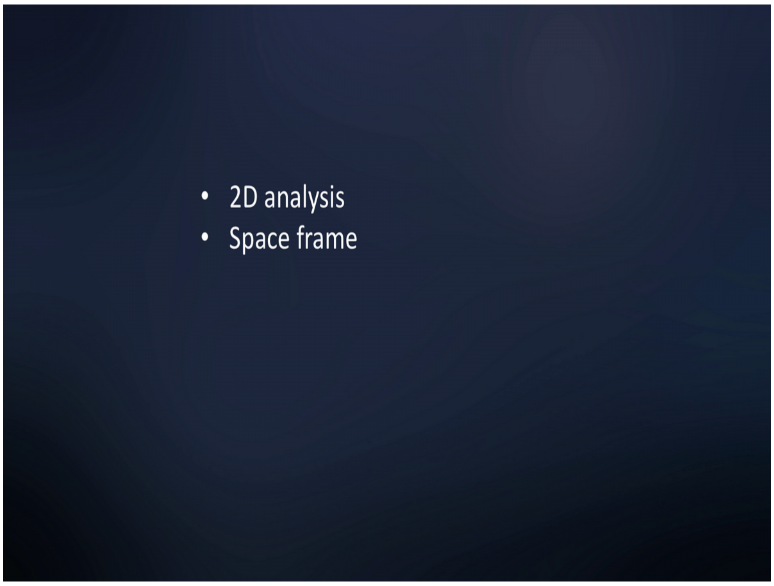


Friends, welcome to the 25th lecture in module 1. In the previous lecture, some 3 d analysis, we discussed about how to evaluate transformation matrix, rotation matrix, how to estimate the direction cosines and the psi angle which are very important for analyzing beam elements or stress elements oriented in space at its arbitrary location with reference to the X, Y, Z axis, let us apply this logic and extend our knowledge of 2 dimensional analysis into 3 dimensional frame structures in the same style.

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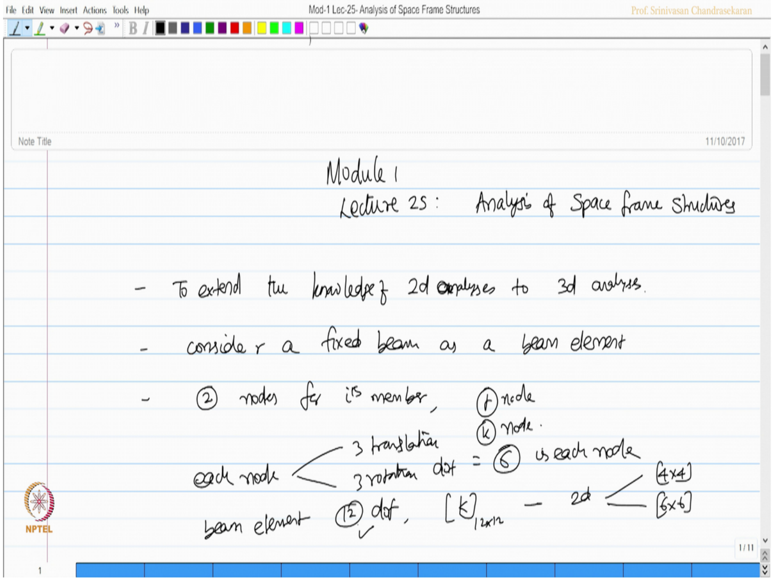


So, the primary objective is to extend the knowledge of 2 dimensional analysis to 3 d analysis. So, we have to follow the same set of expressions and equations with the small modification which will make us to understand very easily and much faster how to use those transformation in a simpler form to do analysis of 3 dimensional space frame structures.

As usual, we will consider a fixed beam for our analysis as a beam element and we all agree that there are 2 nodes for the ith member namely j node and k node each node. Now will have 3 translations and 3 rotation of degrees of freedom making it 6 in each node. So, the beam element now will have 12 degrees of freedom which makes the stiffness matrices size 12 by 12 that is a first difference.

In 2 d analysis, we had seen that the member matrices can be of size 4 by 4, if we neglect excel deformation or 6 by 6, if you include axial deformation for non orthogonal members, in this case, it will be 12 by 12.

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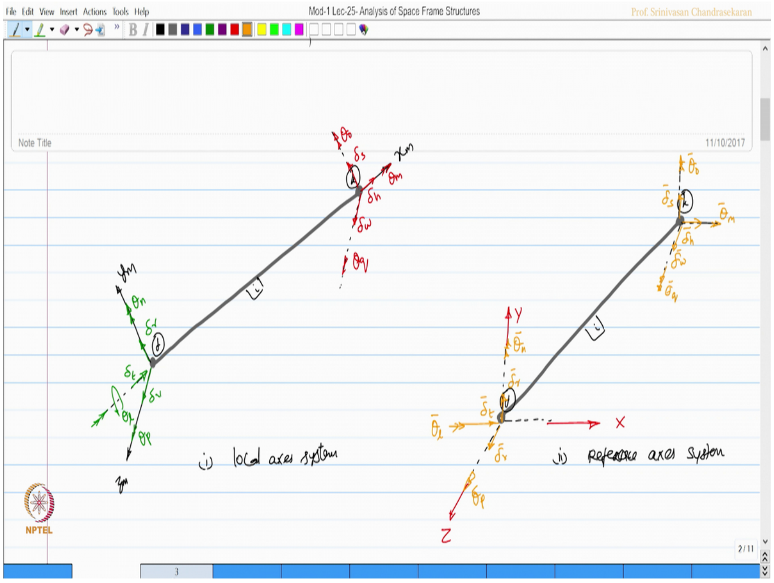


Let us take a simple member arbitrarily oriented arbitrarily oriented with 2 nodes; 2 nodes. Let us mark the nodes as j and k is an ith member that is a member designation.

So, as usual this figure is going to mark the local axes system and this figure is going to represent the reference axis system. So, if this is the member orientation we all know that this is going to be my xm 90 degree anticlockwise to that is our ym and this is zm this of course, is true in both the cases let us mark the axes for marking the degrees of freedom in both joints.

Now, let us mark the translations at the jth end first along x axis. So, let us first mark the axes. So, let us mark the displacement along x, I call this as delta t that is a suffix we are using and along y which is delta r and along z which is delta v, then rotation about this. So, put your thumb towards delta t direction remaining 4 fingers will point the rotation.

So, let us mark it this way as theta l or one can mark with double arrows also l then theta n and of course, theta p. Now let us come to the kth n displacement along x, delta h, along y, delta s and along z, delta w. This what we have used earlier in a derivation also rotation about h 36 axes. So, we call this as theta m and this as theta o and this as theta q this; what we have used in the earlier lecture also, now this is marked as per the local axes system.

I want to mark them for global axes in the similar manner, let me mark the axes first, let say this is my x axis, this is my y axis, this is my z axis, the reference axis system, let us mark the corresponding displacements. So, this is going to be along x which is going to be delta t, but with the bar indicating, it is reference axes system delta t, then delta r, then delta v, then rotations theta l bar theta n bar and theta p bar.

Similarly, here delta h bar, delta s bar and delta w bar, theta m bar, theta o bar and theta q bar. So, we have now 2 sets of degrees of freedom; one expressed in local axes system other expressed in global axes or reference axes system.

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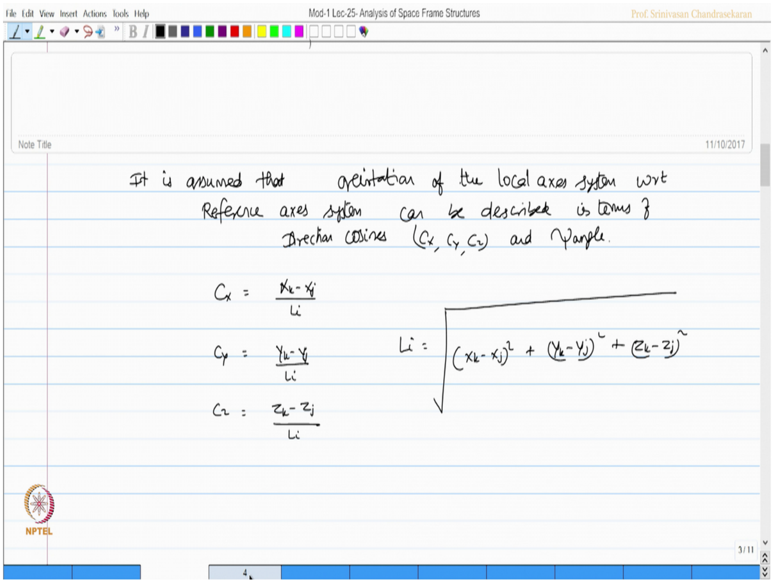


Having said this, it is assumed now that orientation of the local axes system with respect to the reference axes system can be described in terms of the direction cosines C x, C y and C z and the so called psi angle. We also know C x, C y and C z are given by xk minus xj by L i yk minus yj by L i zk minus zj by L i where L i is root of sum of squares of these.

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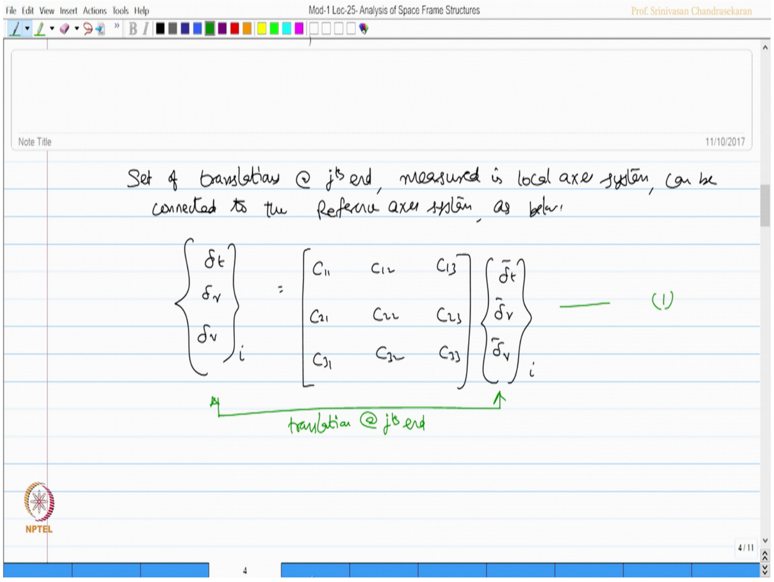


Now, the set of translations at jth end measured in local axes system can be connected to the reference axes system as below, let say the displacements are delta t delta r and delta v for the ith member at the jth node; is it not. So, this can be now connected to C 11, C 1 2, C 1 3, C 2 1, 2 2, 2 3, 3 1, 3 2 and 3 3 of that of delta bar t delta bar r and delta bar v of the ith member.

So, now one can see very clearly here this equation connects the translation at jth end, is it not call this equation number one similarly

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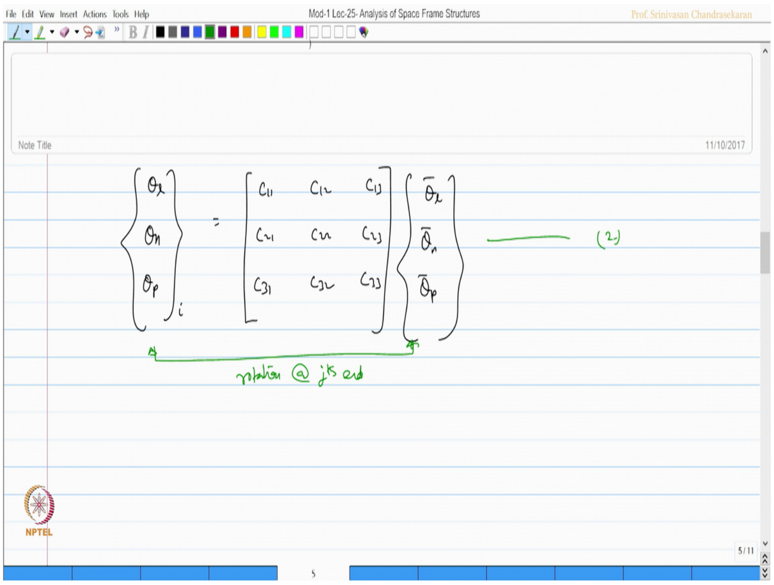

One can also write down for rotations that is theta l, theta n, theta p is now again connected to the global rotations where as these are connected which are nothing, but rotations at jth end equation 2.

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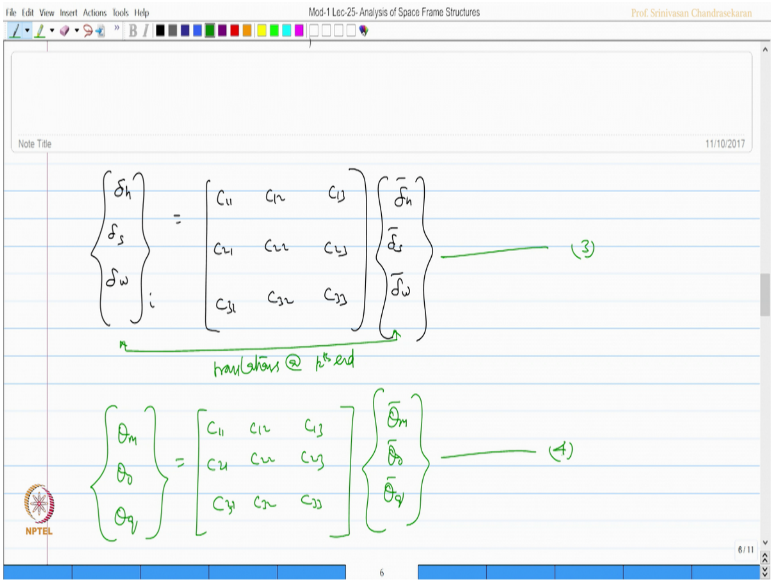

I can also do the same thing for the kth end let us do that quickly delta h, delta s, delta w, is connected to that of the global displacements by this matrix where as these are connecting translations at kth end equation 3. Similarly I can also connect rotations at the kth end which will be theta m, theta o, theta q, theta m bar, theta o bar, theta q bar by a matrix C 1 1, 1 2, 1 3, 2 1, 2 2, 2 3, 3 1, 3 2, 3 3; equation 4.

So, now I can combine all these 4 equations and write in one form. It will be all the 12 displacements.

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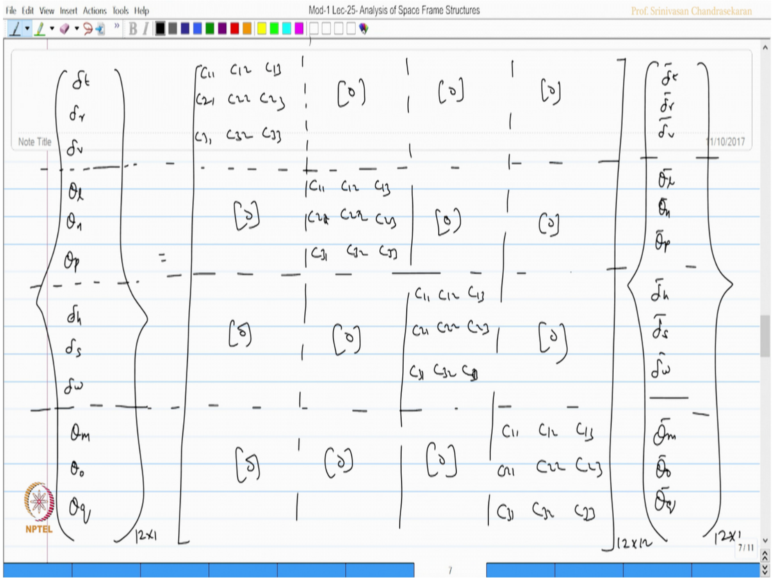

Which will be delta t, delta r, delta v that is translation at jth node theta l, theta n, theta p, rotations at jth node h, s, w, translation at kth node rotations at kth node can be now connected by a transformation matrix to the global which can be delta t bar, delta r bar, delta v bar, theta l bar, theta n bar, theta p bar, delta h bar, s bar, w bar, theta m bar, theta o bar, theta q bar.

Now, each one is a 3 by 3 matrix, here just now we saw that let us divide this also into 3 by 3. So, this is 12 by 1, this is also 12 by 1. So, this has got to be 12 by 12; is it not. So, this is C 1 1, C 1 2, 1 3, 2 1, 2 2, 2 3, 3 1, 3 2, 3 3 and remaining all are 0s. Similarly 1 1, 1 2, 1 3, 2 2, 2 1, 2 2, 2 3, 3 1, 3 2, 3 3, this is the second set which we just now derived, I am just combining all of them.

The third one; 1 2, 1 3, 2 1, 2 2, 2 , 3 1, 3 2, 3 3 remaining all are again 0; the fourth one is the last one; 1 1, 1 2, 1 3, 2 1, 2 2, 2 3, 3 1, 3 2, 3 3 can combine that.

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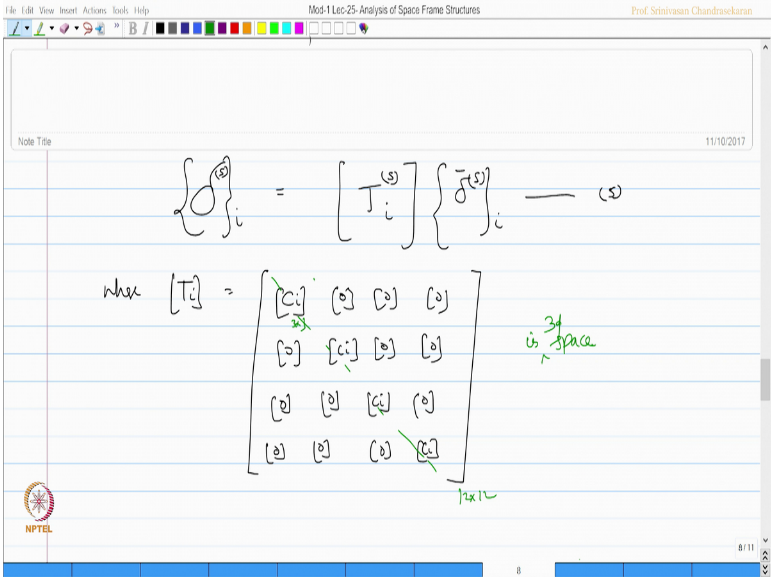

So, now I can write a single expression saying the displacement of the ith member of this space frame is connected to the displacement of the member of the space frame in reference axes system by a transformation matrix which is derived for the space frame for ith member, I call it as question number 5 where ti in simple terms can be expressed as C i 0, 0, 0, 0, C i 0, 0, 0, 0, C i 0 and remaining all are 0s C i.

So, only the diagonal members only the diagonal or C i which are all exactly 3 by 3 that makes 12 by 12 that is the transformation matrix in 3 d space having understood this the transformation matrix has special properties as applied to conventional 2 d analysis.

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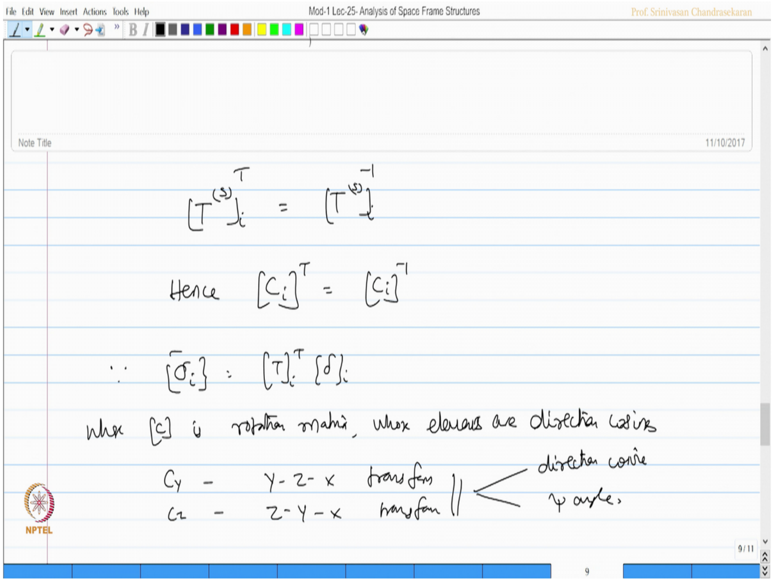


The transformation matrix in space of the ith member transpose will be exactly transformation matrix in space of the ith member with inverse and hence the elements of the transformation matrix whose transpose is also applicable to the same algorithm.

Therefore delta bar I can now be simply said as t transpose of delta i where the C matrix is called the rotation matrix whose elements are the direction cosines. So, C y can be used for Y-Z-X transformation and C z can be used for Z-Y-X transformation which contains the direction cosine and the psi angle correct. So, friends one can also say the equations which are done for 2 d analysis are also valid for 3 d analysis.

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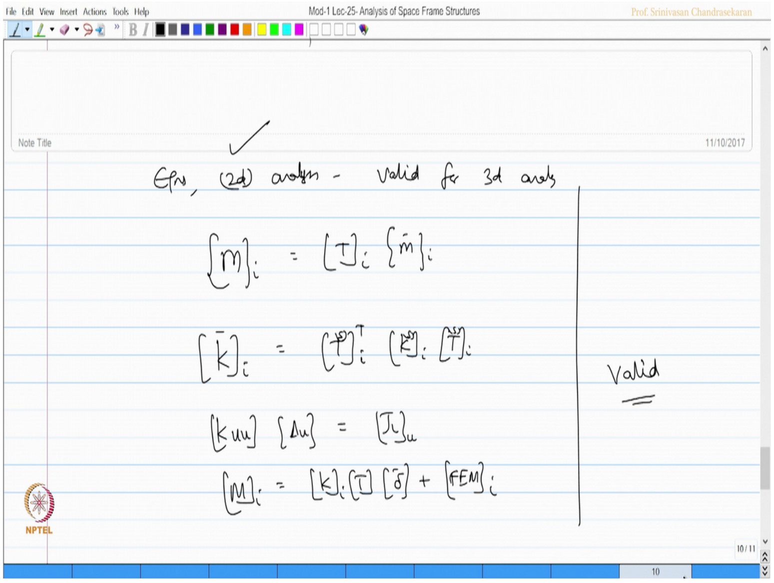


Like the member end reactions will be given by T i of global reactions, if you want to find the global stiffness matrix, I can say it is T transpose k local of T of space this valid similarly if you want to find k unrestrained into delta u unrestrained will give me the joint load unrestrained. So, then end moment of the ith will be k of ith transformation matrix of delta bar plus FEM of the ith member all these relationships are valid which I have been actually derived and applied for a 2 d analysis.

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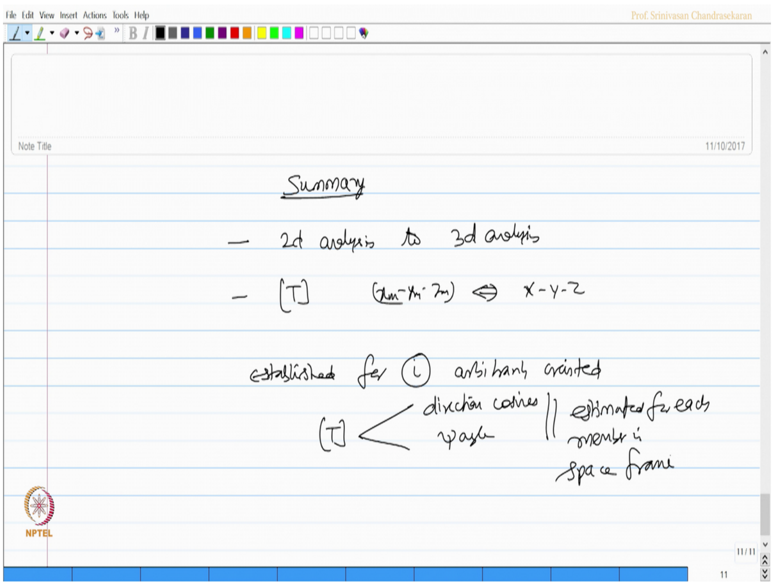

Let us look at the summary; now friends, we are extending the process of understanding 2 d analysis to 3 d analysis, we derive the transformation matrix which connects xm ym zm responses to X-Y-Z system which can be established for any member which is arbitrarily oriented the transformation matrix contains direction cosines and properties of psi angle which need to be estimated for each member in this space frame, we will take of an example in the next lecture and try to solve this problem and apply the concept using computer program.

Thank you very much.