Computer Methods of Analysis of Offshore Structures
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Module – 01
Lecture – 28
3d Truss Analysis

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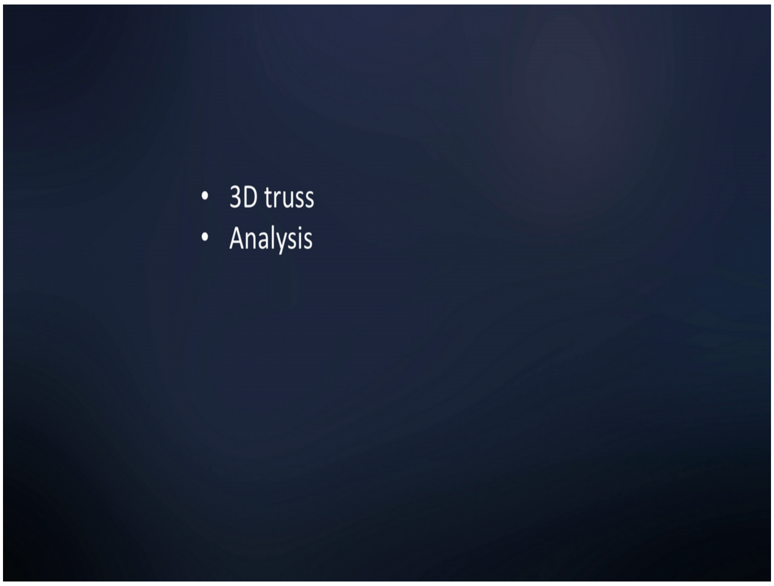

Friends, welcome to the lecture number 28 on module 1. We are going to discuss about 3d truss analysis in this lecture. We will extend the same logic, what we had for 2d analysis of truss members and make the analysis as simple as that to understand 3d analysis concepts. Of course, what are we learnt in the analysis of 3d space frames will be also used in 3d analysis concept of truss members.

Generally, we are having some valid assumptions what we made in the 2d analysis of truss structures.

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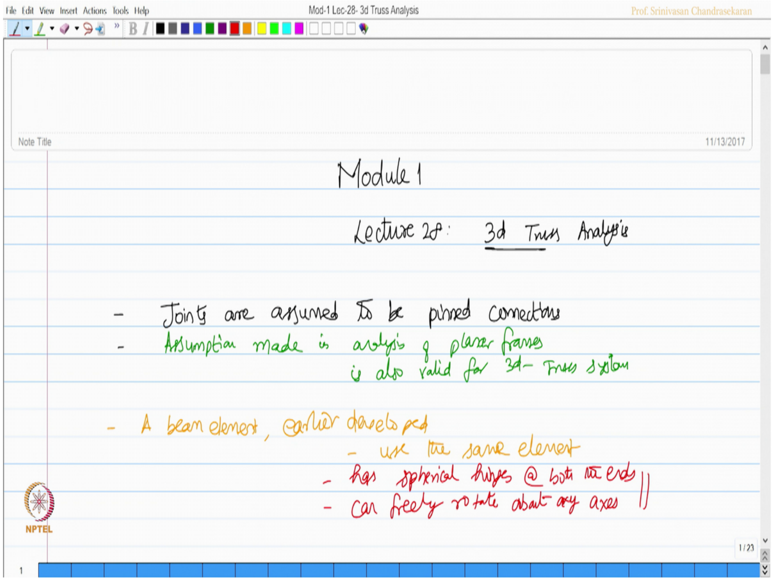

That is; joints are assumed to be pinned connections, which is one of the basic assumption what we made in analysis of planar frames is also valid for 3d truss system; more interestingly we have developed a beam element is it not we have developed a beam element. We will use that element with a small modification; we say that the beam element has spherical hinges at both the ends.

What is the consequence of this assumption? The consequence of the assumption is it can freely rotate about any axes that are the consequence. So, the end rotations will be actually 0 ok.

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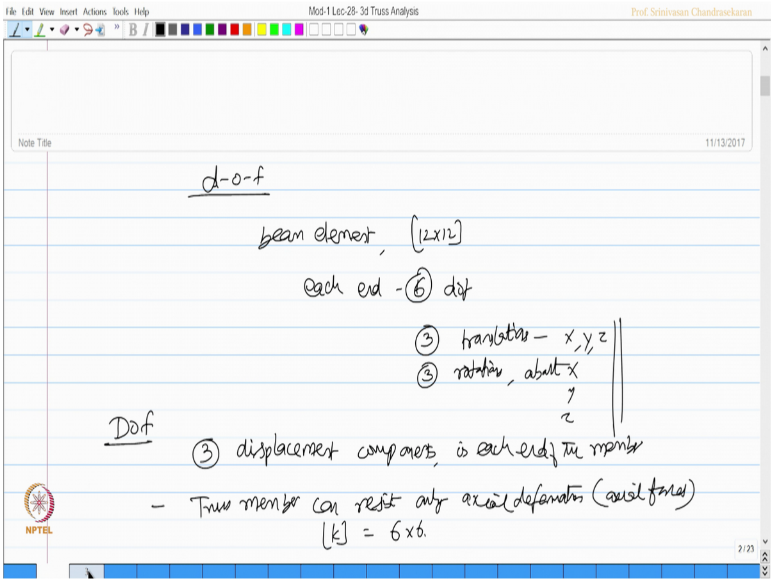

Therefore; what would be the degrees of freedom now we have; earlier for a beam element, if we look carefully we had 12 by 12 matrix that is each end had 6 degrees of freedom, 3 translations is it not along X, along Y and along Z and 3 rotations about X about Y and about Z we had six since we have made a spherical hinges at both the ends the degrees of freedom now are restrained with 3 displacement components.

In each end of the member and most important fact truss members can resist only axial deformation and axial forces so no moments. So, that makes the stiffness matrix of the member as order 6 by 6.

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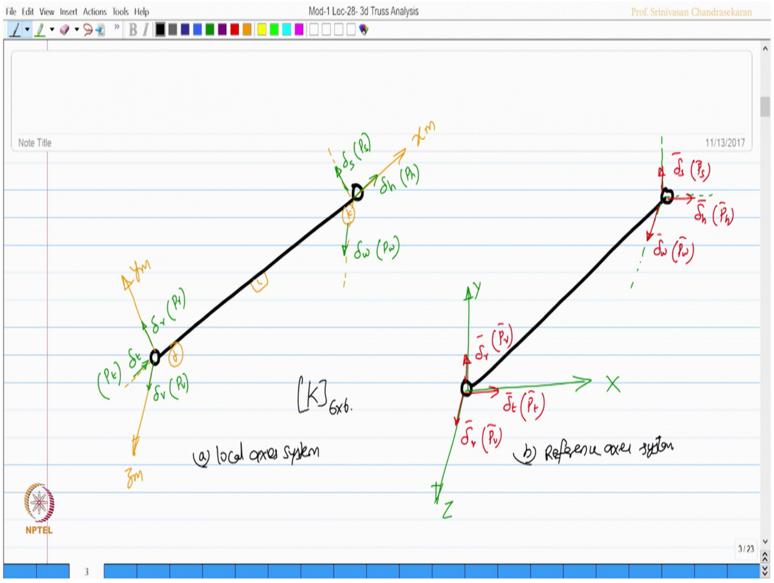

Let us draw a typical truss member arbitrarily oriented with hinges at both the ends.

So, one is local; other is global; I call this as a reference axis system. For the local axis let us mark the axis as x m anticlockwise direction, we know it is y m and we have z m marked here for the member whose j and k ends are marked and the member is i. So, let us have three axes in all the places ok.

Let us now mark the global axes of the system. Let us say this is my X, Y axis and let us say Z axis. Similarly all the three axes at this joint. Let us now mark the degrees of freedom. Let us say along x m I called this as displacement t and this as displacement r and this as displacement v, we are using the same notation if we carefully look at the notations of 2d planar truss analysis except for Z axis displacements all notations are same ok

Similarly, this will be called delta h, this is delta t sorry; delta s and this has delta w which has same as the beam element, what we used in the earlier derivation. So, let us also mark the corresponding force for our understanding axial forces I can call this as P t this as P r, this is as P v, this as P s, this as P h and this as P w.

Let us now correspondingly mark the global displacement degrees of freedom we call this as delta t bar, this is going to be delta r bar and this is delta v bar and as simple as that delta h bar delta s bar and delta w bar let us also mark the corresponding forces. So, P t bar, P v bar, P r bar, P h bar, P w bar and P s bar.

So, P indicates the axial forces in the respected degrees of freedom at j and k ends. And the degrees of freedom are marked which are similar to exactly the beam element except that the ends of the members are spherical hinges which can rotate about any axis freely therefore, there are no moments about any of these axis so; obviously, we now we realize that a stiffness matrix will have size 6 by 6.

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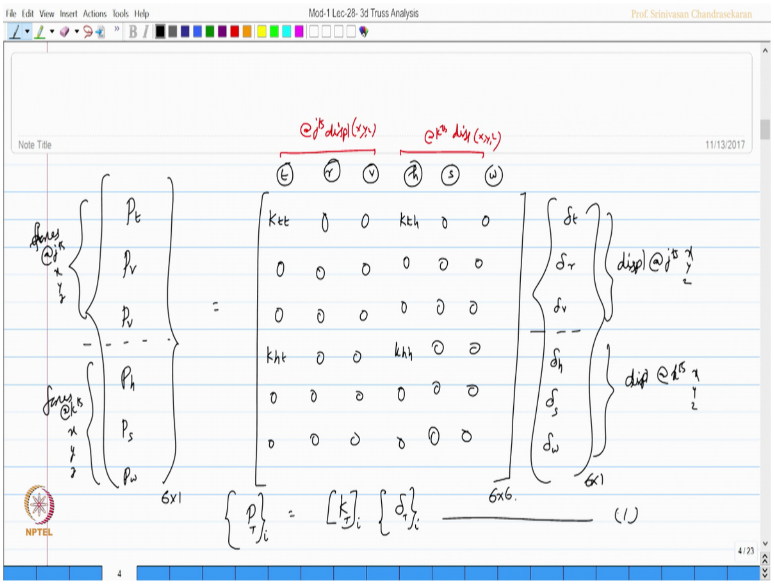

So, now let us say I want to connect my displacement or the forces P t, P r, P v that is at the jth end P h, P s and P w this is displacement; this is displacement sorry forces at jth end along x y and z and this is forces at kth end along x y and z which will be equal to the stiffness matrix connecting the displacements delta t, delta r, delta v, delta h, delta s delta w. So, these are displacements at jth end at x y and z these are displacements at kth end at x y and z connecting will be a stiffness matrix this is 6 by 1; this is 6 by 1.

So, this has got to be 6 by 6 the labels could be t r v h s and w. So, you know this will be at jth end displacements is it not and this will be at kth end placements along x y and z along x y and z so; obviously, for a truss member you know this value will be k t t and r and v contribution will be 0, then you will have k h t and these two will be 0 similarly for the column h you will have kth this will be 0 and k h h this will be 0 remaining all member for a truss element will be 0 in this stiffness matrix. So, that is my stiffness I can now say the force local of the truss member of ith member is given by stiffness matrix of the truss member of the ith member multiplied by the displacement local of the truss member of the ith member only equation number 1 ok.

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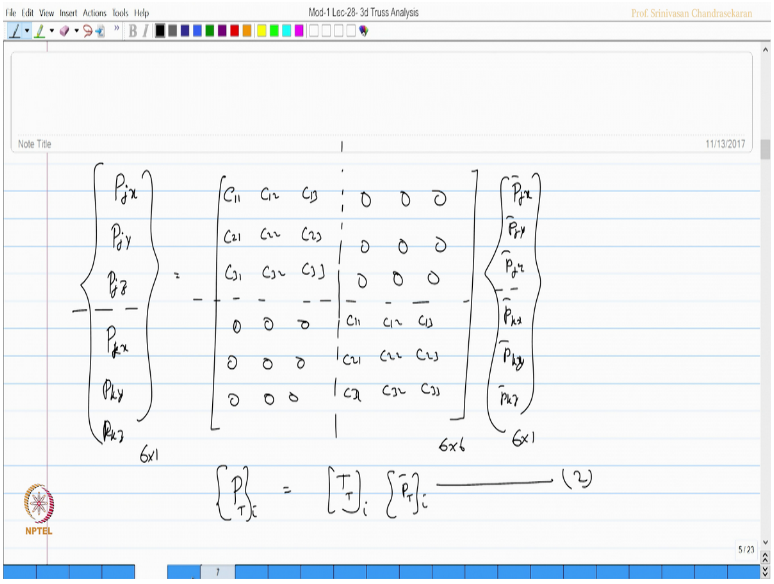

Now, I want to find out the forces; which will connect the global forces. So, let us say P at jth end along x, P at the jth end along y, P at the jth end along z. Similarly P at the kth end along x, P kth end along y, P at the kth end along z which will be now connected by some matrix which will connect the local to the global. So, P bar j x, P bar j y, P bar j z, P bar k x, P bar k y, and P bar k z ok.

So, now this is again 6 by 1, this is 6 by 1 this has got to be 6 by 6, but interestingly I can now divide this at 4 quarters. So, this will be C 1 1 C 1 2 C 1 3 2 1 2 2 2 3 3 1 3 2 and 3 3 remaining this particular part will be 0 and by symmetry this will also be 0 and this will repeat as 1 1, 1 2, 1 3 2 1 2 3 3 1 3 2 and 3 3 which is the order of same style as that of beam element in a 3d analysis except that in a beam element it was 12 by 12 where as truss element it is 6 by 6.

So, now I can write now this as P of the truss element local will be given by t of the truss element transpose matrix of i and by connecting all the truss element ith member equation number 2.

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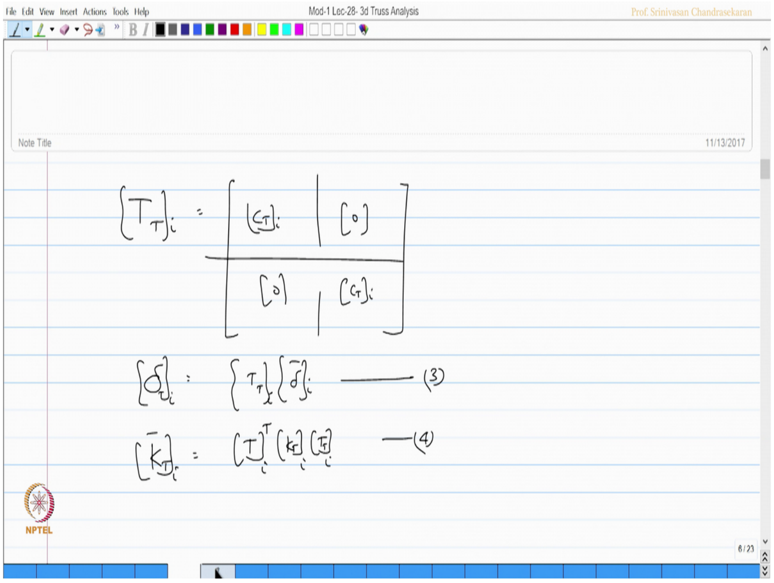

I can also expand this matrix as T of the truss member of every member simply can be said as C T i 0 0 and C T of the ith member is it not.

So, now with this algorithm we can always say the displacement of the truss member simply is T of the truss member with the displacement bar of the ith member, K transpose of the truss member can be also said as T transpose. This is K bar the truss member the T transpose K of the truss member with the T on the truss member this is for every member. So, I can call this equation number 3, 4 ok.

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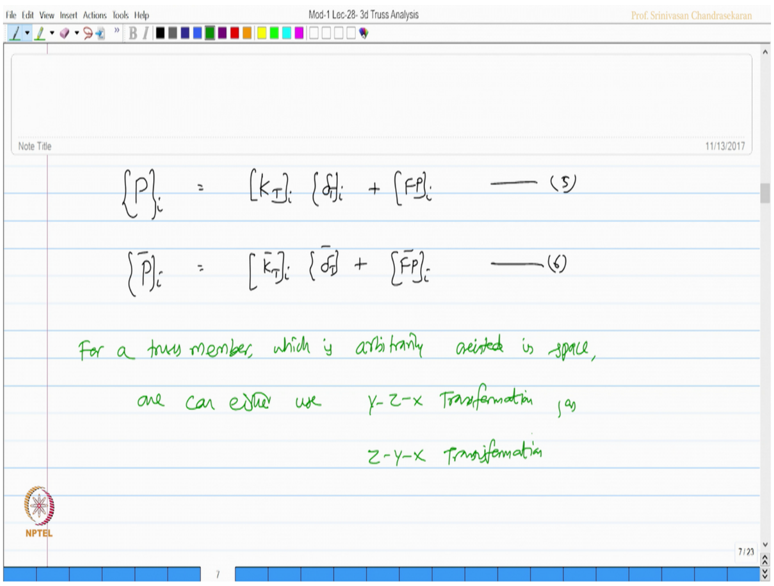

Further we can always say the force in the ith member is K multiplied by the displacement of the ith member plus any joint load applied on the ith member equation 5.

If you want to find the forces in the global degrees of freedom, this can be simply said as K bar of the truss member of the ith member multiplied by displacements of the truss member in the global degrees of freedom plus transformation of the loads in the global degrees of freedom. So, friends; very interestingly for a truss member which is arbitrarily oriented in space one can either use Y Z X transformation or Z Y X transformation .

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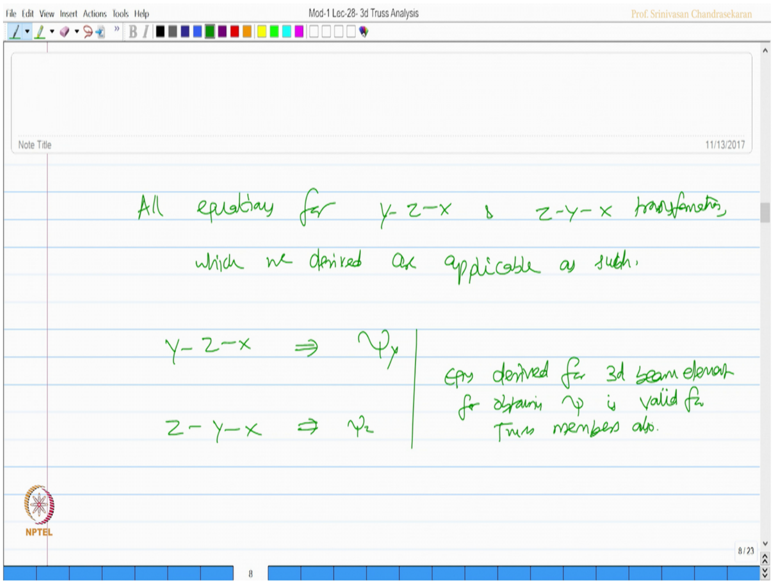

So, therefore, all equations for Y Z X transformation and Z Y X transformations, which we derive are applicable as it is; without any change no change is required straight away you can apply. So, using Y Z X will enable you to compute the psi by angle using Z Y X will enable you to compute the psi Z angle. So, the equations derived for 3d beam element for obtaining the psi angle is for truss members also there is only one important catch.

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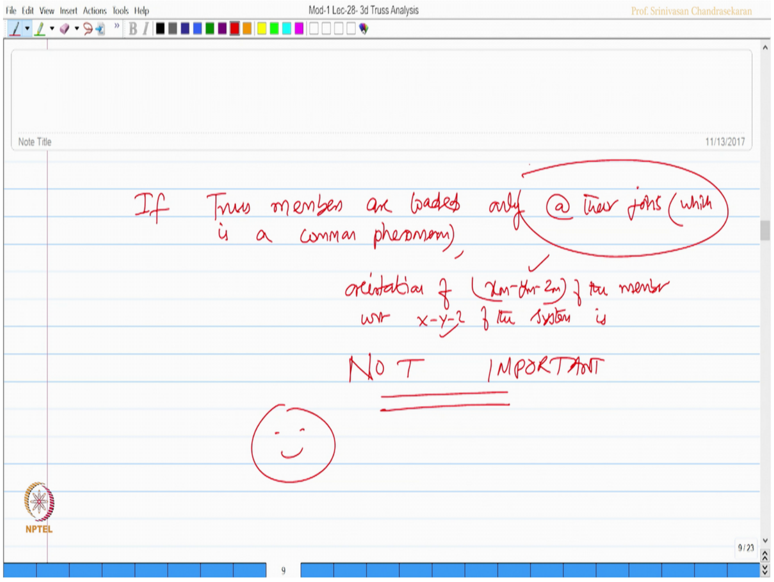

If truss member are loaded only at their joints which is usually a common phenomenon; only joint loads are considered no member loads. So, usually a common phenomenon in that case orientation of the local axes of the member with respect to the reference axis of the system is not important that is a very great relaxation we have that is a very great relaxation.

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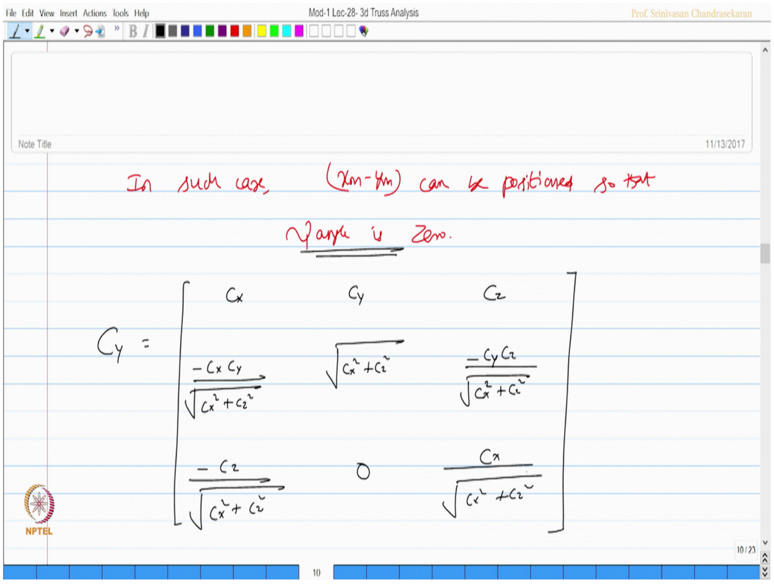

We have we do not have to bother about the orientation of the local axes, with the global axes provided; if the loads are only appeared at the joints. In that case the x m, y m axis can be positioned. So, that the psi angle is practically 0.

So, if you agree upon this statement, then C y which was derived earlier can be now modified for the truss member very easily by taking psi angle as 0. So, C y is given by C x C y C z minus C x C y by root of C x square plus C z square root of C x square C z square minus C y C z by root of C x square C z square minus C z by root of C x square C z square 0 C x by root of C x square C z square that is C y.

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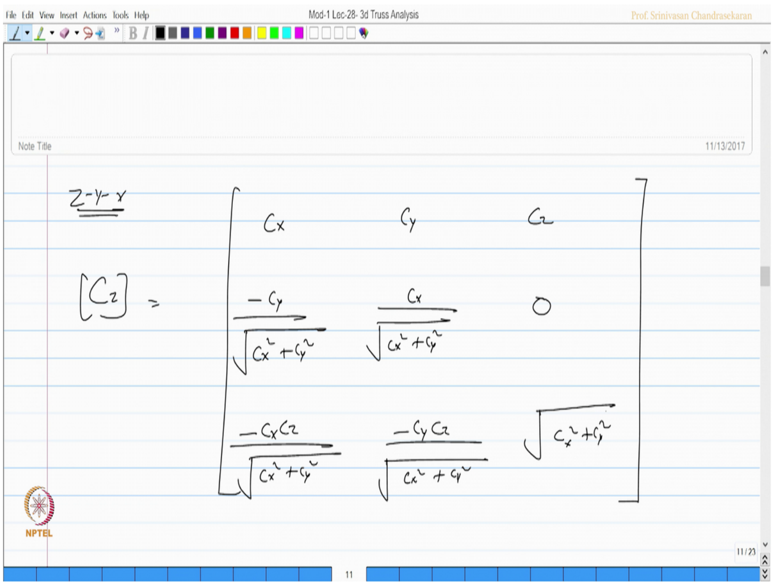

You can also write C z for Z Y X transformation as C x C y C z. We derived this equation earlier; except that we are substituting psi angle as 0 in the whole equation minus C y by C x square plus Cy square C x by C x square C y square 0 minus C x C z by C x square Cy square minus C y C z by C x square Cy square root of C x square plus C y square.

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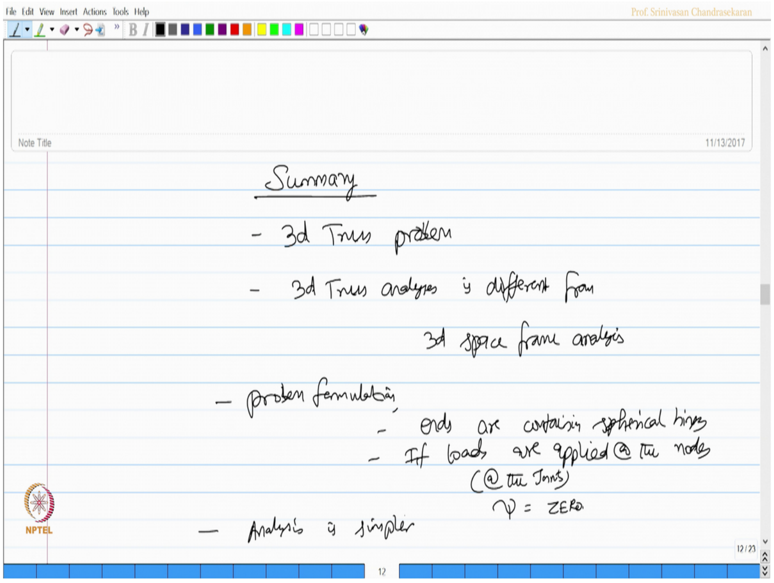

So, friends we can make use of the existing derivations and try to solve a simple 3d truss problem. So, we have learnt how a 3d truss analysis is different from 3d space frame analysis. In the problem formulation we have assumed that ends are containing spherical hinges, that is the first assumption we made, second is if loads are applied at the nodes, that is at the joints we do not have to bother about the psi angle keep this as 0 directly do the transformation matrix.

So, analysis is simpler. I believe that you will able to understand and use the same program with a small modification of a 3d element and solve a truss problem and try to understand the application of this programming for the truss problem easily. So, friends we will have another one more lecture in module 1, where we are going to talk about the special elements that is non prismatic members. How they can be handled using what is called as sub structure technique which we will discuss in the successive lectures.

Thank you very much and bye.