Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture – 29
Special Elements

Friends we have been discussing about the stiffness method of structural analysis applicable to 2 dimensional members; orthogonal, non orthogonal beam elements and truss elements. We extended same logic to understand the response behavior of finding end reactions and displacements of 3d structural members, space frames and space truss members using the transformation matrix and the transformation procedures; however, in certain situation, we may also encounter structural members with varying cross section with non uniform moment of inertia etcetera. We can call them as special elements.

Now, special elements are treated by different techniques by different authors in various text books, but I would take you in a different path; we would also treat the special elements in the same style as we have been treating the conventional beam elements or truss elements.

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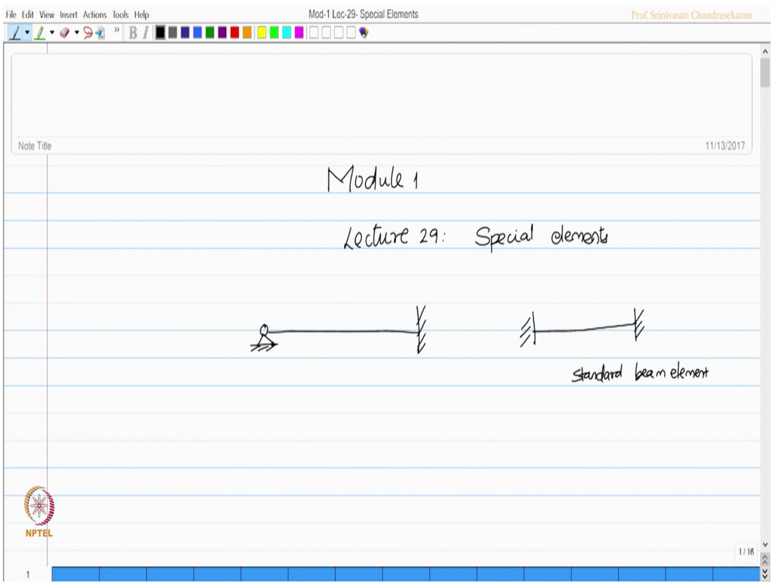

So, how do we handle them? So, this lecture 29 is going to discuss about the formation of stiffness matrix for special elements. Let us take for example; I have an element with one end fixed and other end hinged. So, for my standard beam element my standard beam element has both ends fixed; we have also discussed the beginning of this module, how to handle special elements with varying moment of inertia and cross section.

But here the support conditions are different. So, what do we are going to do is? We are going to convert this special member to a conventional member with the procedure what we have followed so far.

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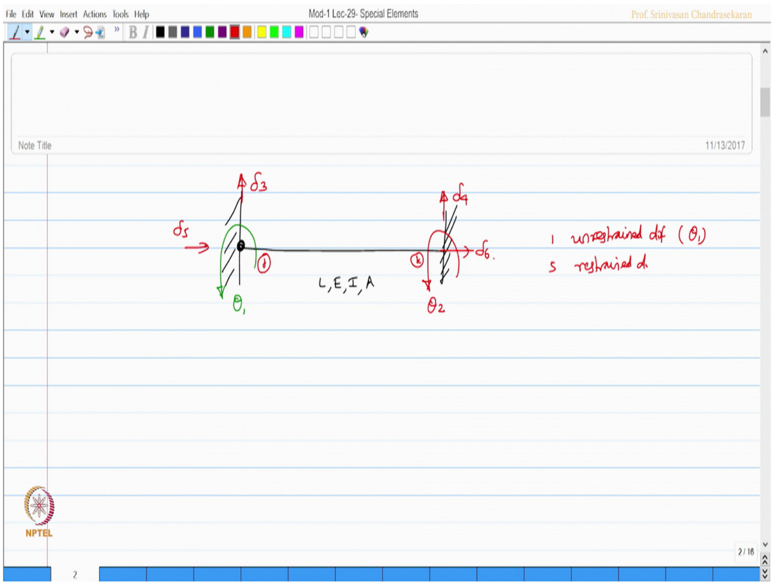

Interestingly, my problem now becomes, a fixed beam with hinge introduced whose length modes of elasticity, moment of inertia and area of cross section are uniform, but I would call this as unrestrained degrees of freedom remaining all or restrained degrees of freedom.

So, I am following the same logic as we did for the beam element and marking the degrees of freedom as we have been doing for a conventional beam element. So, I have got the j and k end of the member. So, now, the beam has got one unrestrained degree, which is theta 1 and 5 restrained degrees, which is theta 2 onwards till delta 6. So, therefore, this analysis will be conventionally dealt in the same style as we have been doing for a beam element.

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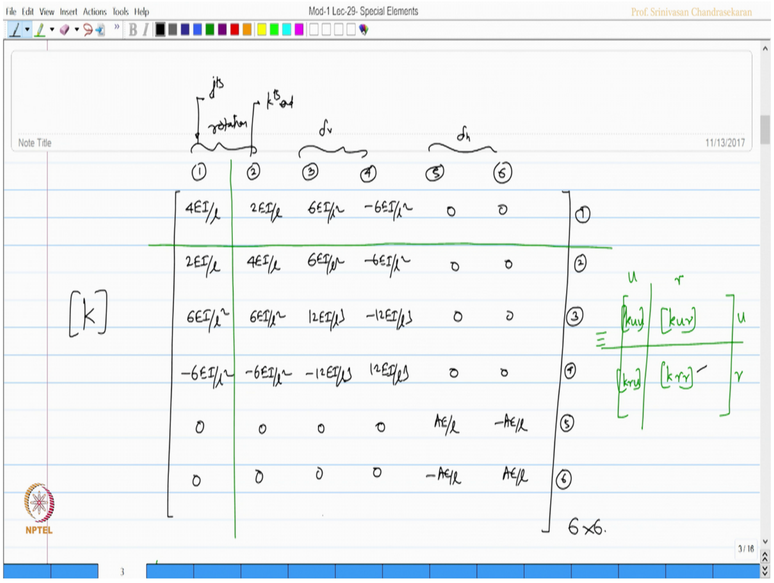

We know for a beam element the stiffness matrix can be written as 6 by 6 with degrees of freedom as you see here which is going to be one rotation and the jth end, kth end. Vertical displacement jth end kth end axial deformation jth end kth end ok.

This is rotation this is delta v, this is delta h and this is for jth end this is for kth ends so on. Similarly; so we all know we remember this derivation from the first lecture this is simply 4 E I by l this is 2 E I by l this is 6 E I by l square this is minus 6 E I by l square and these two are 0. Similarly this is 4 E I by l 2 E I by l 6 E I by l square minus 6 E I by l square and again 0. This will be 6 E I by l square 6 E I by l square 12 E I by l cube minus 12 E I by l cube and 0, this will be minus 6 E I by l square minus 6 E I by l square minus 12 E I by l cube and 12 E I by l cube again 0.

These elements of the stiffness matrix are 0 whereas; this is A E by l minus A E by l minus A E by l and A E by l. This is a standard stiffness matrix for a fixed beam which we have already have with us, but applying this to this problem, one of the degrees of freedom is unrestrained, so let me partition this. I have partition at one, now I can say this matrix is equivalent to a stiffness matrix which is partition causing the sub matrix. So, this is unrestrained degree, this restrained degree; similarly unrestrained and restrained.

So, this becomes K unrestrained row unrestrained column, this becomes K unrestrained row restrained column, this becomes K restrained row unrestrained column and this becomes K restrained row restrained column. All are individually sub matrices. So, I am going to pick up these matrixes separately and write down in the next slide. So, for example, K u u will be equal to 4 A E by l and so on. So, K u u is going to be 4 E I by l. So, I can always find K u u inverse as l by 4 E I I also need K r r, K u u is actually 1 by 1 k r r is this matrix which is 5 by 5 this matrix ok.

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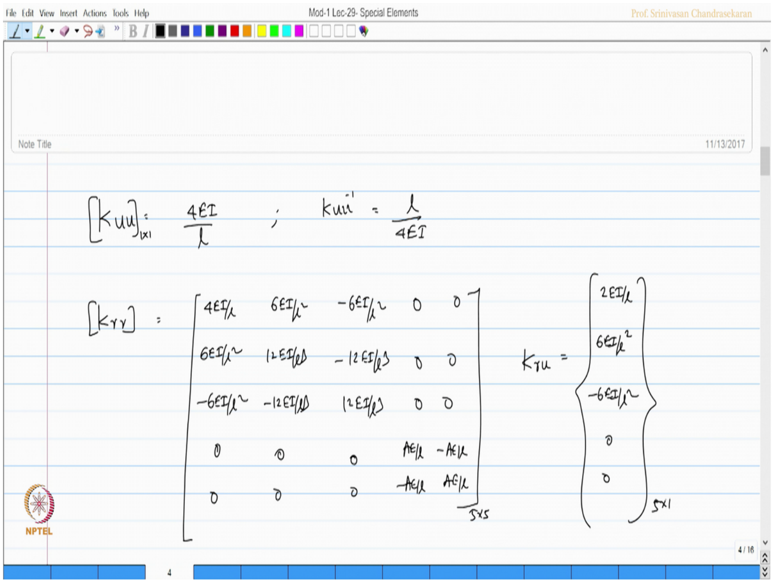

Let us enter that 4 E I by l 6 E I by l square minus 6 E I by l square and so on. So, 4 E I by l 6 E I by l square minus 6 E I by l square 0 0. Similarly the next column will be starting from 6 E I by l square 12 E I by l cube minus 12 E I by l cube 0 0. Next column is minus 6 E I by l square minus 12 E I by l cube 12 E I by l cube 0 0. Next column is about three zeros and A E by l. So, I have simply plugged out the K r r matrix from the K matrix which is 5 by 5 matrix ok.

I also now require K r u; K r u is this matrix which I am looking into this column which is actually this vector which is 5 by 1. Let us pick up that vector which is 2 E I by l 6 E I by l square minus 6 E I by l square 0 and 0 which is 5 by 1. Let us also have K u r;

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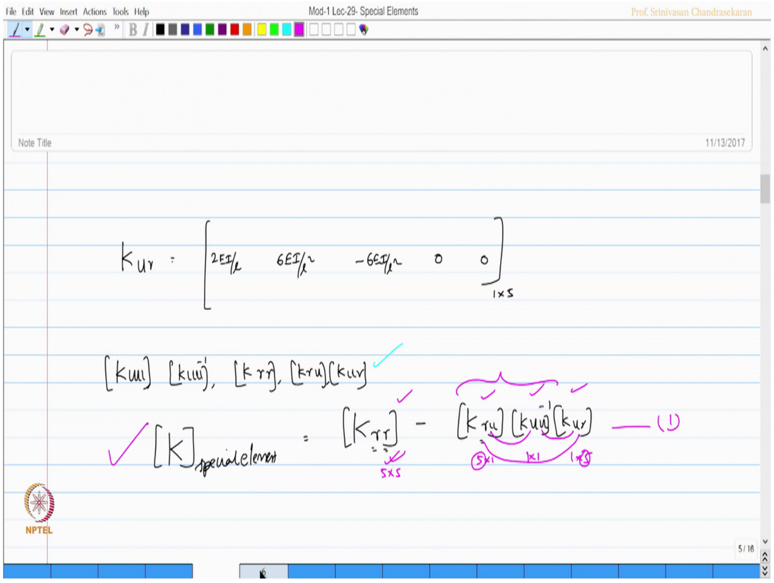

K u r is unrestrained row restrained column which is going to be this particular matrix; this matrix which I am writing here 2 E I by l 6 E I by l square minus 6 E I by l square 0 and 0 which is going to be 1 row into 5 columns. Now I have K u u K u u inverse K r r K r u and K u r I have all of them.

These are all sub matrices which I have estimated and evaluated from the original stiffness matrix which is available here correct which is a standard procedure. We are not doing any derivation new we have already derived this matrix for a simple fixed beam. What I have done is? I have considered this as my unrestrained degree of freedom in the whole derivation; that is all I have done. So, a partition the matrix and got these sub matrices. Once I get this I want to now know, what is my k matrix of the special element? That is the bother issue.

K at a special element can be easily given by K r r minus K r u K u u inverse and K u r we can see here r r r u and u u and u and r and r I will get K r r look at the size K r r is 5 by 5 K r u is 5 by 1 this is 1 by 1 and this is 1 by 5 I get again 5 by 5. So, let us do this. So, I have all these matrices with me. Let us substitute in this equation one and try to find K of the special element. So, let us first do this operation K r u into K u u inverse.

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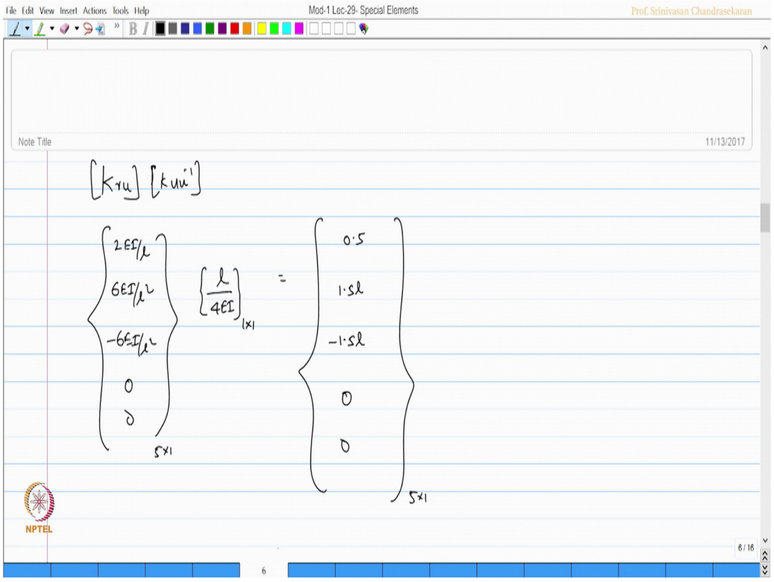

So, K r u into K u u inverse let us do that K r u is 2 E I by l 6 E I by l square minus 6 E I by l square 0 0 which is actually 5 by 1 K u u inverse is l by 4 E I which already we have with us is 1 by 1. So, now, I will get a vector when I multiply these two which will be 5 by 1, so which is going to be 0.5 1.5 l minus 1.5 l 0 0.

The next step is I want to multiplied this answer with K u r. I multiply this with K u r, where K u r actually is a size of 1 by 5. So, I will ultimately get 5 by 5. So, let us do that.

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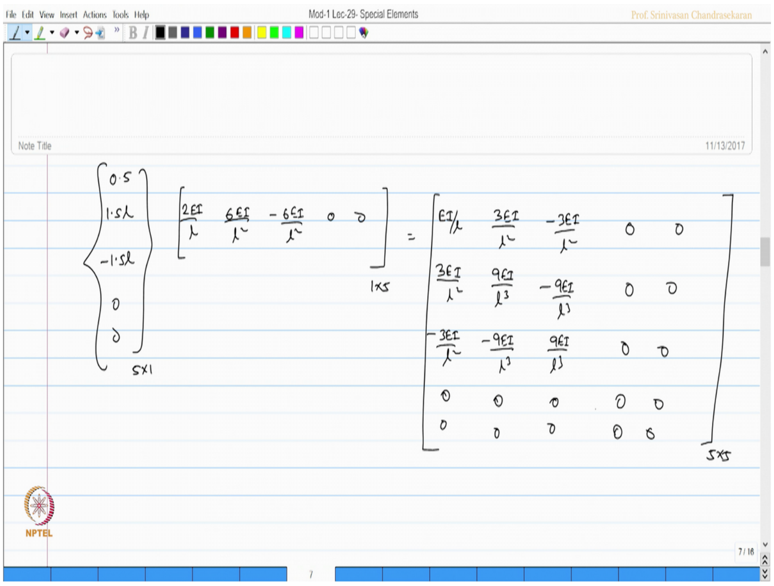

So, this is going to be 0.5 1.5 l minus 1.5 l 0 0 multiplied by 1 by 5 which is K u r which is 2 E I by l 6 E I by l square minus 6 E I by l square 0 0. So, this is 5 by 1 is 1 by 5 I will ultimately get a 5 by 5 matrix; let us find out this first, then we will do the other one which is going to be so, this value is going to be E I by l 3 E I by l square minus 3 E I by l square 0 0 3 E I by l square 9 E I by l square minus this is l cube minus 9 E I by l cube 0 0 minus 3 E I by l square minus 9 E I by l cube 9 E I by l cube 0 0 and this will be all zeros. Now my K special element is actually equal to K r r minus this product which this value.

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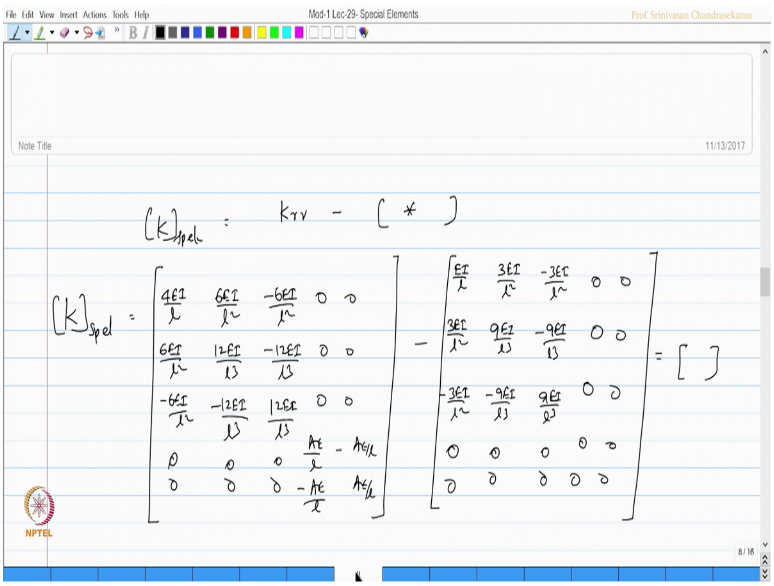

So, let us write down that. So, K special element is going to be K r r minus this matrix. So, K r r is 4 E I by l 6 E I by l square minus 6 E I by l square 0 0 6 E I by l square 12 E I by l cube minus 12 E I by l cube 0 0 minus 6 E I by l square minus 12 E I by l cube 12 E I by l cube 0 0.

These elements are 0 whereas; this is A E by l minus A E by l minus A E by l and A E by l. I subtract from this; this value which I am writing here as E I by l 3 E I by l square minus 3 E I by l square 0 0 3 E I by l square 9 E I by l cube minus 9 E I by l cube 0 0 minus 3 E I by l square minus 9 E I by l cube 9 E I by l cube 0 0 and these are zeros as you see. Now the result is going to be the matrix which we want which I write here.

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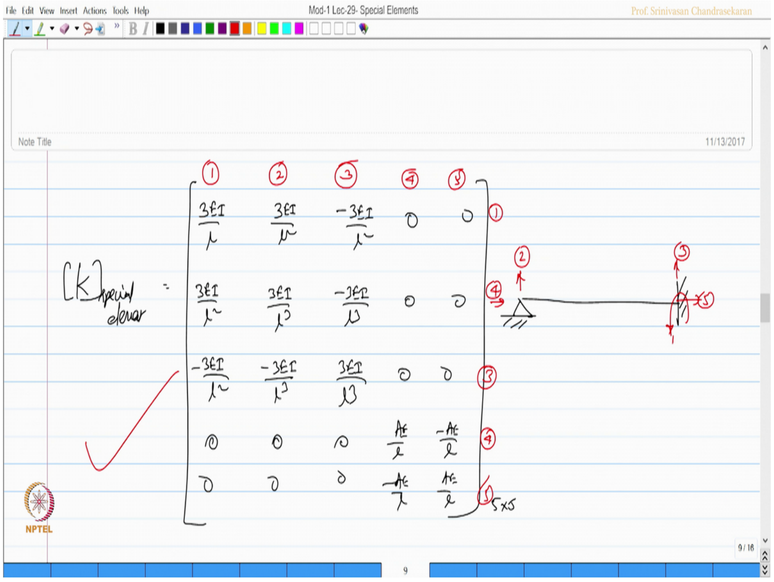

So, K special element is actually 3 E I by l 3 E I by l square minus 3 E I by l square 0 0 3 E I by square 3 E I by l cube minus 3 E I by l cube 0 0 minus 3 E I by l square minus 3 E I by l cube plus 3 E I by l cube 0 0. And these elements are 0 whereas; this is A E by l minus A E by l minus A E by l A E by l.

So, that is my 5 by 5 matrix which now resembles a beam of this style. We have just got the degrees of freedom which are restrained like this. So, let us mark 1 is absent. So, I will call as 1, this as 2, this as 3, this as 4 and 5 is as 5 I mark the labels here this is going to be 1, 2, 3, 4 and 5 which is my special element. So, friends please understand, I can always use a modification in the stiffness method to derive the stiffness matrix of any element of my choice with where I boundary conditions or support conditions given in the problem. So, that is very interesting; we are using the same procedure as developed for a conventional beam element to derive stiffness matrix of a special element.

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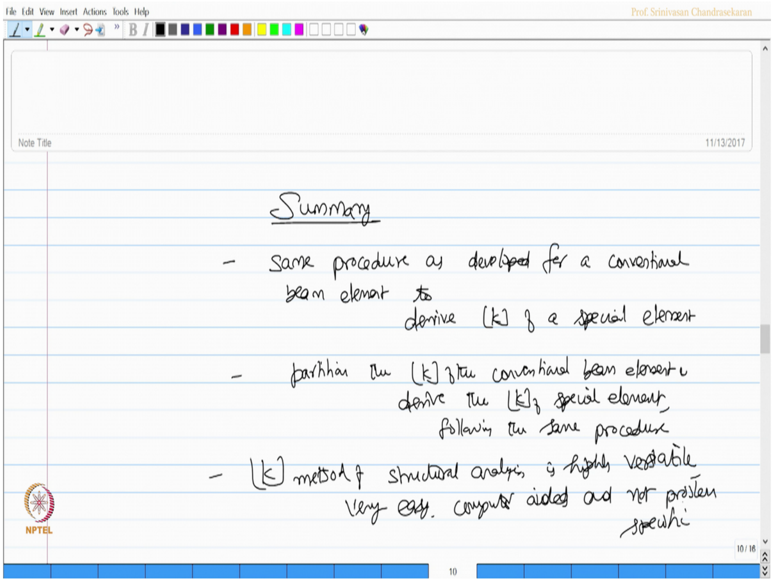

The method is very simple; partition the matrix of the conventional beam element, and derive the stiffness matrix of the special element following the same procedure as we have discussed. So, stiffness method of structural analysis is highly versatile, and I should say very easy and computer aided and not problem specific. It is highly a generic method; I believe that you have enjoyed the lectures of module 1. We will now do one more lecture on explaining how this special element can be solved using the conventional stiffness method procedure. And also give you the computer program for that so, that that completes entire discussion on module 1. So, friends we will have left only one more lecture and module 1 where I am going to solve and example problem using special element for given beam element.

Thank you very much and bye.