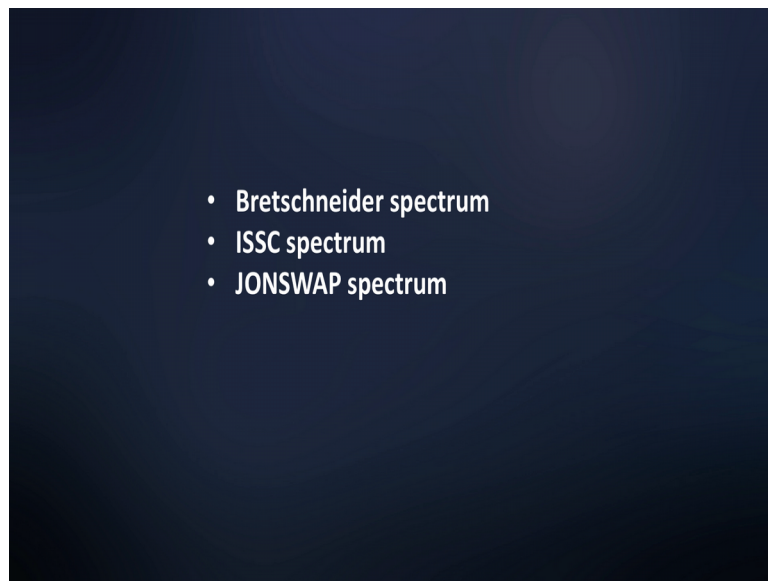


Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module – 02
Lecture – 08
Wave Spectra (Part – 2)

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(2) Bretschneider spectrum

$$S^+(\omega) = 0.16 \omega_s^4 H_s \left(\frac{\omega_s^4}{\omega^5} \right) \exp \left\{ -0.675 \left(\frac{\omega}{\omega_s} \right)^{4.7} \right\}$$

where ω_s = significant wave frequency

(3) International Ship Structure Congress (ISSC)

$$S^+(\omega) = 0.1107 H_s \left(\frac{\bar{\omega}^4}{\omega^5} \right) \exp \left\{ -0.4427 \left(\frac{\omega}{\bar{\omega}} \right)^{4.7} \right\}$$

$\bar{\omega}$ = significant wave frequency

NPTEL logo is visible in the bottom left corner of the whiteboard interface.

The second spectrum which is commonly used is Bretschneider spectrum, it says the spectral density is given by $0.1687 H_s \omega_s^4 \omega^{-5}$, above 4 exponential minus $0.675 \omega_s \omega^{-5}$ where ω_s is significant wave frequency, the third spectrum commonly used is developed for ship structures which is international ship structure congress, which is ISSC spectrum which says the spectral density is given by $0.1107 H_s \bar{\omega}^4 \omega^{-5}$ exponential minus $0.4427 \bar{\omega} \omega^{-5}$ where $\bar{\omega}$ is significant frequency in this expression.

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$$S^+(\omega) = \frac{\bar{\alpha} g^2}{\omega^5} \exp \left\{ -1.25 \left(\frac{\omega}{\omega_0} \right)^4 \right\} \gamma a(\omega)$$

$$a(\omega) = \exp \left\{ - \frac{(\omega - \omega_0)^2}{2 \bar{\sigma}^2 \omega_0^2} \right\}$$

$$\bar{\sigma} = \begin{cases} 0.07 & \text{for } \omega \leq \omega_0 \\ 0.09 & \text{for } \omega > \omega_0 \end{cases}$$

$$\bar{\alpha} = (3.25 \times 10^{-3}) H_s^2 \omega_0^4 (1 - 0.287 \ln(\gamma))$$

ω_0 : significant wave freq
 γ : peakedness parameter (1 to 7)
 H_s : significant wave height (m)

The fourth one which is again used commonly is Jonswap spectrum, which says the spectral density is $\bar{\alpha} g^2 \omega^{-5} \exp \left\{ -1.25 \left(\frac{\omega}{\omega_0} \right)^4 \right\} \gamma a(\omega)$ where $a(\omega)$ is exponential minus $\frac{(\omega - \omega_0)^2}{2 \bar{\sigma}^2 \omega_0^2}$ $\bar{\sigma}$ is 0.07 for ω less than ω_0 0.09 for ω greater than ω_0 . $\bar{\alpha}$ is parameter $3.25 \times 10^{-3} H_s^2 \omega_0^4 (1 - 0.287 \ln(\gamma))$ of the gamma.

Where ω_0 is significant wave frequency and this is called Peakedness parameter which varies anywhere from 1 to 7 and of course, H_s is significant wave height in meters.

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```

MATLAB CODE FOR WAVE SPECTRA PLOT:
%%
%%-----
%%JONSWAP spectrum
%%Wave height is taken as 5 m
%%Time period is taken as 10 seconds
hs=5; %wave height in m
t=10; %Time period in seconds
g=9.81; %gravitational constant
w=0.0001:0.1; %frequency is the varying component
n=length(w);
w0=(2*pi)/t;
alpha=1.25*(10^-3)*(hs*t)^2*(w0^4)/(1-0.287*log(w));
for i=1:n
    IF w(i)<w0
        sigma(i)=0.07; %spectral width parameter
    else
        sigma(i)=0.09;
    end
    x(i)=-(w(i)-w0)^2/(2*(sigma(i))^2*(w0)^2);
    y(i)=1.25*(w(i)/w0)^(-4);
    a(i)=exp(x(i));
    b(i)=exp(y(i))*sigma(i)*alpha*lg^2;
    p(i)=(1/(w(i)^5));
    s(i)=a(i)/w0;
end
%%PM spectrum
%%Wave height is taken as 5 m
%%Time period is taken as 10 seconds
%%Mean wind speed is 20 m/s
hs=5; %wave height in m
t=10; %Time period in seconds
g=9.81; %gravitational constant
w=20; %mean wind speed in m/s
w0=(2*pi)/t;
w=0.0001:0.1; %frequency is the varying component
n=length(w);
for i=1:n
    x(i)=1.25*(w(i)/w0)^(-4);
    a(i)=exp(x(i));
    b(i)=1/(w(i)^5);
    p(i)=0.0051*a(i)*b(i)*lg^2;
    s(i)=w(i)/w0;
end

```

So, friends please pay attention to the programming code available on the screen now, this programming code gives you plots of different spectra for Jonswap, PM spectrum and. So, on the coding is available on the screen you can type it back and run it in mat lab you will be able to plot the PM spectrum.

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```

%%
%%-----
%%Modified PM spectrum
%%Wave height is taken as 5 m
%%Time period is taken as 10 seconds
hs=5; %wave height in m
t=10; %time period in seconds
w0=(2*pi)/t;
w=0.01:3; %frequency is the varying component
n=length(w);
for i=1:n
    y(i)=(-1.25)*((w(i)/w0)^(-4));
    a(i)=exp(y(i));
    b(i)=(w0^4)/((w(i))^5);
    s2(i)=0.3125*((hs^2)*a(i)*b(i));
    p2(i)=w(i)/w0;
    i=i+1;
end

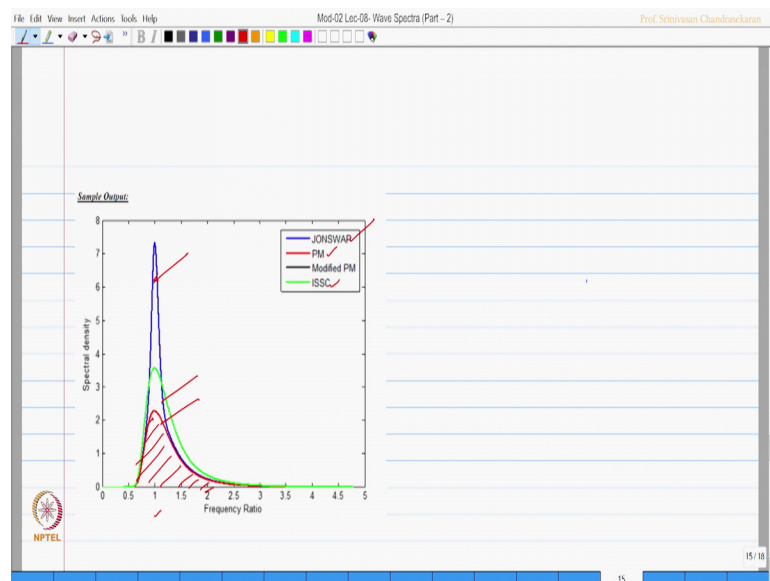
```

You can also plot the ISSC spectrum and compare them for a specific mean wind speed of 20 meter per second, significant wave height of 5 meter and time of 10 seconds whose typical plot looks like this.

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```
File Edit View Insert Actions Tools Help Mod-02 Lec-08- Wave Spectra (Part - 2) Prof. Srinivasan Chandrasekaran
%%plots
plot(p,s,'b','linewidth',2); %johnswap spectrum
hold on;
plot(p1,s1,'r','linewidth',2);%Modified PM spectrum
hold on;
plot(p2,s2,'k','linewidth',2); %Bretschneider spectrum
hold on;
plot(p3,s3,'g','linewidth',2); %ISSC spectrum
xlabel('Frequency Ratio');
ylabel('Spectral density');
title('WAVE SPECTRA (Mean wind speed=20m/s, Wave Height=5m, Time
period=10s)');
```

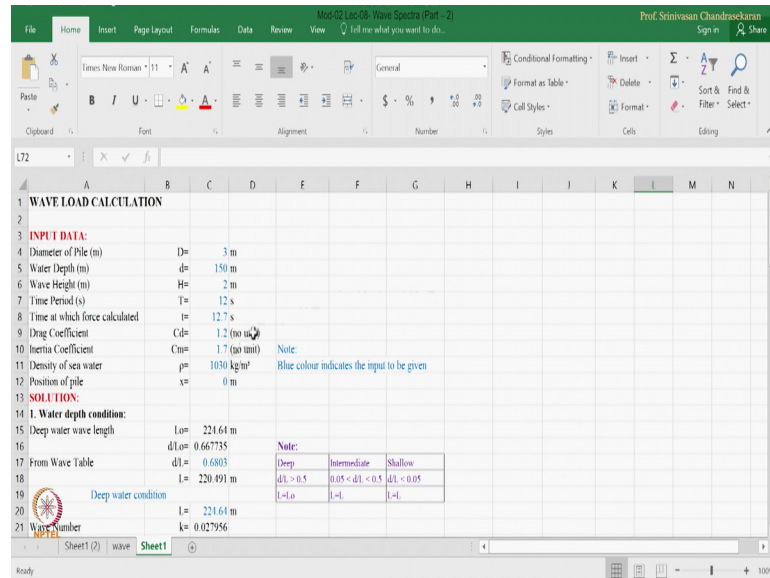
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So, one can see here the Jonswap the Jonswap spectrum which is indicated in blue color, compact PM structure indicated in red color which is also further modified with black which has got the same variation as the top red and which is compared with ISSC spectrum, you will see Jonswap spectrum has a very narrow band concentrated at this specific frequency ratio, but the spectral density of modify PM spectrum is much larger compared to that of ISSC and Jonswap.

So, friends one can use any of these spectra to estimate the wave forces in a given system. So, we will take up an example and see how a wave forces can be computed in a given system. The simple excel sheet I want to you to pay attention on this.

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Whatever you see in blue color they are all input to be given for example, here is three meter 150 you can vary this, it calculates some of the values based upon 2π by for example, $C/20$ which is $L/2\pi$ by L we know it is a wave number we already gave the equation 5. So, it calculates a wave number wave frequency is 2π by $t/0.7$ you can see here is wave period ok. So, 2π by $t/0$ and so on.

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21 Wave Number $k = 0.027956$

22 Wave Frequency $\omega = 0.523333$

24 2. Calculation of Wave Force

25 Horizontal water particle velocity $u(x,t) = \frac{\omega H}{2} \frac{\cosh(ky)}{\sinh(kd)} \cos(kx - \omega t)$

26

27 Horizontal water particle acceleration $\dot{u}(x,t) = \frac{\omega^2 H}{2} \frac{\cosh(ky)}{\sinh(kd)} \sin(kx - \omega t)$

28

29 Morison's equation $dF_T = \frac{1}{2} C_{DP} \rho |u| dz + C_{M} \rho \frac{\pi D^2}{4} \dot{u} dz$

30 In the presence of current

31 current (m/s) 1

32 Force calculation interval 10 m

33 $y = d + z$

z	y	velocity	acceleration	Drag force	Inertia Force	Total Force	Drag force	Total Force
0	150	0.4894	-0.0973	4441.03	-12040.62	-7599.59	41128.96	29083.34
-10	140	0.3701	-0.0736	2539.87	-9105.68	-6565.81	34804.17	25698.50
-20	130	0.2799	-0.0557	1452.94	-6887.02	-5434.07	30373.23	23486.21
-30	120	0.2118	-0.0421	831.54	-5210.11	-4378.58	27224.35	22014.24
-40	110	0.1603	-0.0319	476.27	-3943.05	-3466.78	24959.34	21016.28
-50	100	0.1214	-0.0241	273.16	-2986.16	-2713.01	23313.98	20327.82
-60	90	0.0920	-0.0183	157.01	-2264.18	-2107.14	22109.67	19845.49

So, I want him calculate the wave force based on the water particle kinematics horizontal velocity and acceleration, then to compute the force we use Morison equation in the presence of current which is one meter per second.

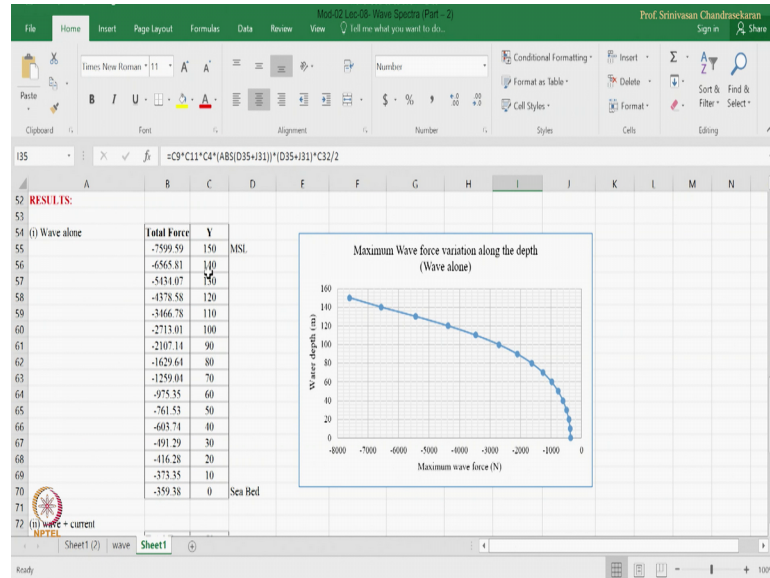
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z	y	velocity	acceleration	Drag force	Inertia Force	Total Force	Drag force	Total Force
-70	80	0.0699	-0.0139	90.66	-1720.30	-1629.64	21223.53	19503.23
-80	70	0.0533	-0.0106	52.71	-1311.75	-1259.04	20569.80	19258.06
-90	60	0.0409	-0.0081	31.02	-1006.38	-975.35	20087.86	19081.48
-100	50	0.0317	-0.0063	18.65	-780.17	-761.53	19734.54	18954.37
-110	40	0.0250	-0.0050	11.60	-615.34	-603.74	19479.06	18863.72
-120	30	0.0203	-0.0040	7.62	-498.91	-491.29	19299.60	18800.69
-130	20	0.0171	-0.0034	5.45	-421.73	-416.28	19181.09	18759.36
-140	10	0.0154	-0.0031	4.37	-377.72	-373.35	19113.68	18735.96
-150	0	0.0148	-0.0029	4.05	-363.43	-359.38	19091.82	18728.39

So, if you look at different values of Z and Y velocity acceleration the drag force and the inertia force are computed total force is added and total force is computed, one can see any specific value here which is taken from the code.

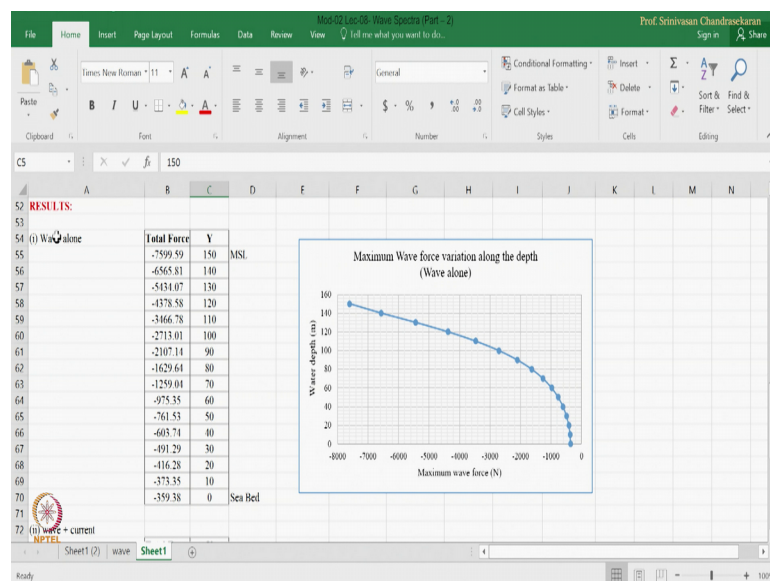
The code is available on the screen you can always reproduce it in a excel sheet and try to plot the variation.

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If you see if the variations are plotted from the sea bed towards the mean sea level, from the sea bed towards the mean sea level because the water depth is 150 meter you can see here the water depth is 150 meter. So, I am trying to plot from the sea bed towards the mean sea level, mean sea level is 150.

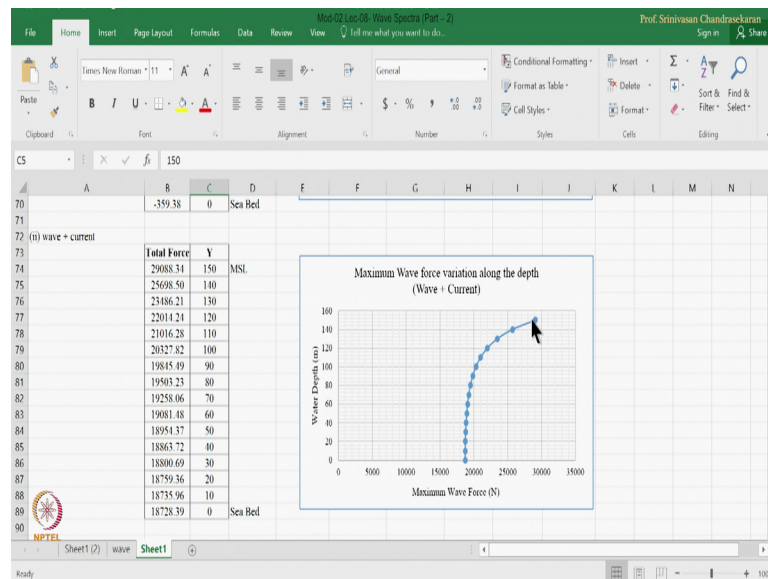
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So, if you take only about the waves there is a typical variation you see.

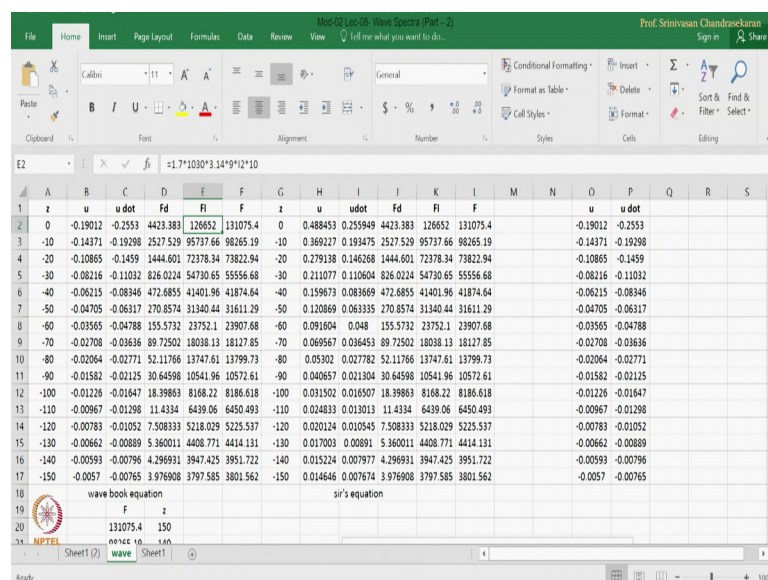
If you plot adding current wave there is a variation you can see.

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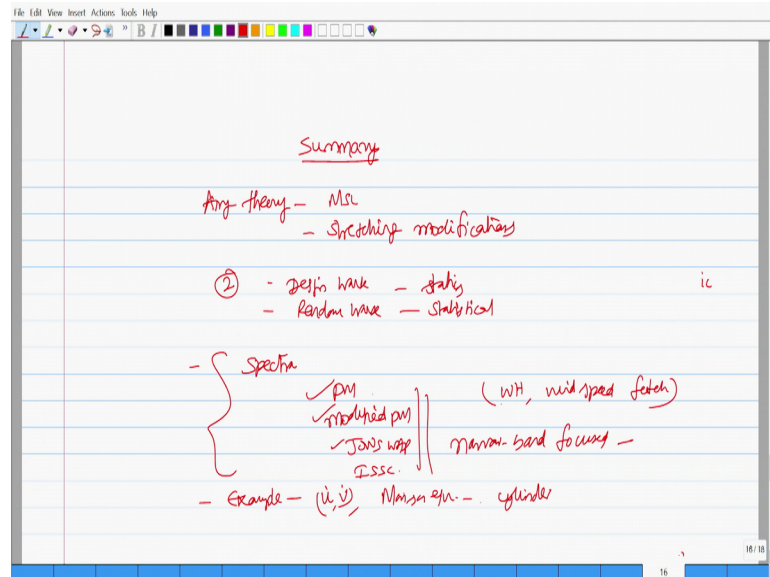
So, that is a very interesting code we have in simple terms, the detailed calculations of doing this has also been indicated in the worksheet here, is also being given in the worksheet here.

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So, the comparison of the plot with wave and current and with wave alone is also shown here. So, simple coding which helps us to formulate the problem and try to find force in a single cylinder which can be used. So, that is the variation of the spectrum.

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So, friends let us look at quickly the summary what we learned in this lecture we understood Airy's theory has limitations in terms of mean sea level you can use stretching modifications suggested by various researchers to include the variable submergence effect one can do two approaches for design. One can be a single design wave analysis one can also do random wave analysis, one is statics no dynamics, other is statistical analysis you will be using wave scatter diagram one can also find the forces using various spectra available in the literature.

We have seen Pierson Moskowitz spectrum, modified Pierson Moskowitz spectrum Jonswap spectrum and ISSC spectrum. We have compared we said that modified Pierson Moskowitz spectrum gives a broader area, Jonswap is a narrow band focused spectrum which can be used for application of offshore structures in specific terms. We have also seen an example how to compute the forces on a given member using horizontal water particle velocity in acceleration then using the Morison equation, which is used to calculate forces on a given cylindrical member whose coding is also available. We also gave you the computer program to estimate the spectra and plot them and compare them for a given wave height for a given wind speed and fetch conditions.

Thank you very much.