Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 02
Lecture - 13
Dynamic Analysis - 2 (Part - 2)

I can now write the control equations for finding the mode shape and frequency.

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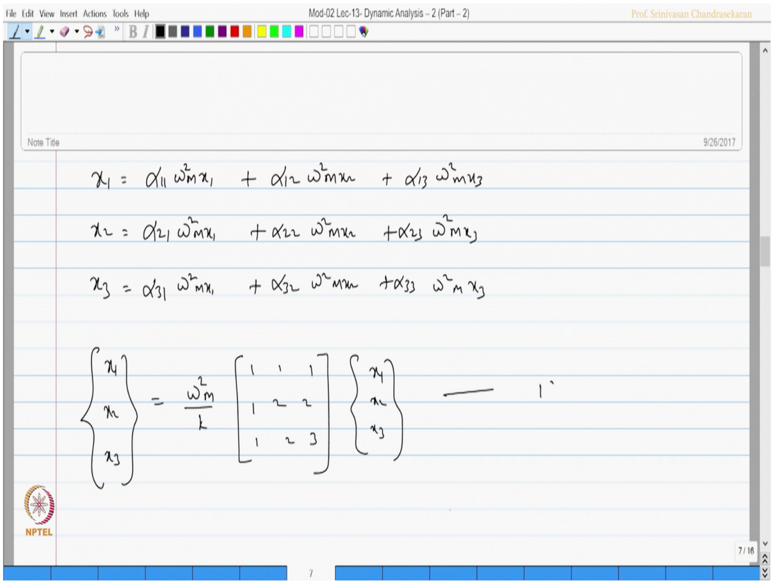

x 1 x 2 x 3 is actually equal to alpha 11, alpha 21, alpha 31 plus alpha 12 alpha 22 alpha 32 alpha 13 alpha 23 and alpha 33 that is first row first column first row second column first row third column and so on, multiply omega square m x 1 omega square m x 1 omega square m x 1. So, this becomes omega square m x 2, m x 2, m x 2; omega square m 3 omega square m x 3 omega square m x 3.

I can now write this in a matrix form x 1 x 2 x 3 as omega square m by k of 1 1 1, 1 2 2, 1 2 3 that is my alpha matrix with m. I will multiply it by x 1, x 2, x 3 - equation 1. Now you can see in this equation the vector x 1, x 2, x 3 are present both in left hand side and right hand side. So, the scheme becomes iterative I assume the value of this vector substitute and see what value I am getting back if the assumed value and the obtained value are same then the solution is converged.

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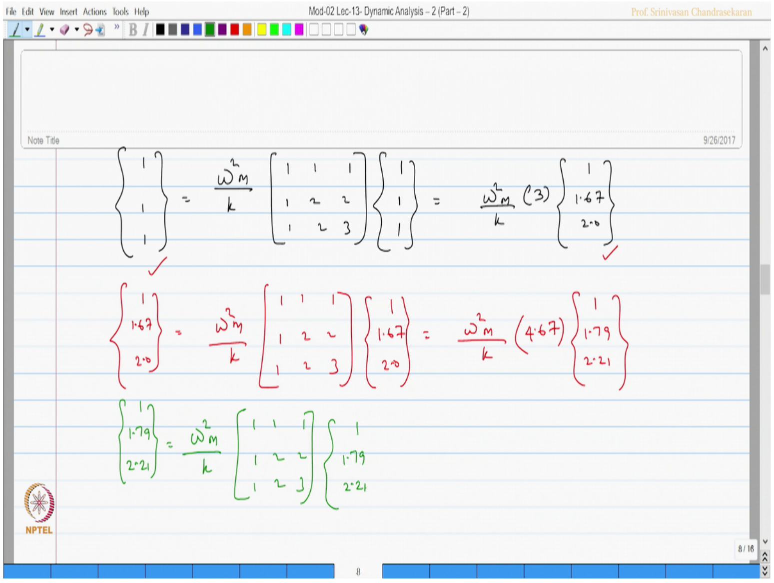

Let us start with the first mode shape zero crossing let us take it as all 1 1 1 which will be omega square m by k times of 1 1 1, 1 2 2, 1 2 3 of 11 1 let us multiply and see what happens. So, omega square 1 m by k I take a multiply 3 out if we do that I get a vector as 1, 1.67 and 2.0. I started with 1 1 1, I got 1, 1.67, 2 so no convergence let us start the second iteration with 1, 1.67 and 2.0 as omega square m by k 1 1 1, 1 2 2, 1 2 3 multiplied with 1, 1.67 2.0 which gives me omega square m by k 4 0.67 is the multiplier I get 1, 1.79, 2.21.

Again this converges not happened let us take the next iteration 1, 1.79, 2.21 is omega square m by k 1 1 1, 1 2 2, 1 2 3 of 1, 1.79, 2.21 which becomes omega square m by k. Now the multiplier is 5, if I do that I get this value as 1, 1.8, 2.24. I do one more iteration 1, 1.8, 2.24 is equal to omega square m by k 1 1 1, 1 2 2, 1 2 3 multiplied by 1, 1.8, 2.24 which will give me omega square m by k the multiplies 5.04 and I get a vector as 1, 1.801 and 2.25.

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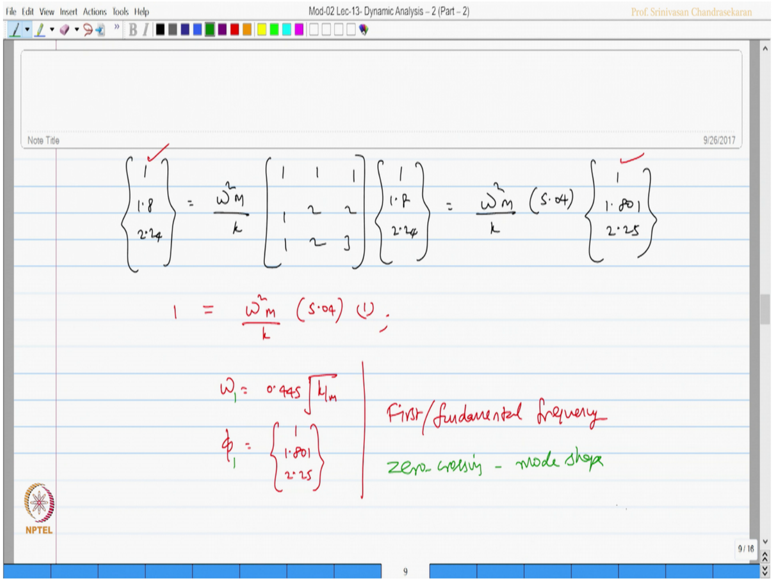

So, they are more or less converging once they are converging let us write down the equality 1 is equal to omega square m by k 5.04 of 1 which implies that omega is actually equal to 0.445 square root of k by m and the corresponding mode shape is 1, 1.801, 2.25. So, this is my first or fundamental frequency. Why fundamental frequency? I think you will you have to guess this, this is because the mode shapes has zero crossing. So, it is the first mode shape. So, I should say now omega 1 and phi 1.

So, I can also obtain the fundamental frequency and mode shape using influence coefficient method as omega 1 and phi 1.

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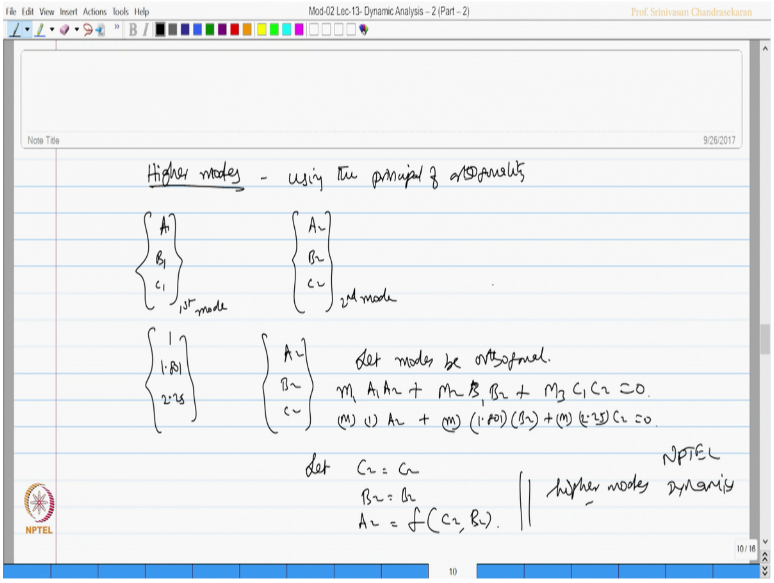

I can also obtain the higher modes using the principle of orthogonality. So, I can also get the higher modes using the principle of orthogonality, I will quickly explain you let us say A 1, B 1, C 1 is the first mode vector and let A 2, B 2, C 2 be the second mode vector in other case is going to be 1, 1.801 and 2.25. Let us do not know the second mode take it as A 2, B 2 and C 2.

So, let modes be orthogonal if they are orthogonal then m 1 A 1 A 2 plus m 2 B 1 B 2 plus m 3 C 1 C 2 will be 0. So, m in our case is 1 value a 1 in our case is 1 and a 2 we do not know plus m 2 now cases m and 1.801 and B 2 plus m 2.25 and C 2 is set to 0. So, I will get an equation in A 2 B 2 C 2. So, let C to be C 2, B 2 be B 2 and express A 2 as a function of C 2 and B 2 and then apply the algorithmic principle and try to obtain the higher modes. I am not solving the higher modes in this particular lecture you can please refer to the NPTEL notes on dynamics or the textbooks which you have referred in the last lecture on dynamics written by me and you can compute the higher modes.

So, now we have learned the second method that is influence coefficient method which gives me the value of fundamental frequencies and mode shapes and higher modes as well. So, now, let us look at the third method which is given by Stodola.

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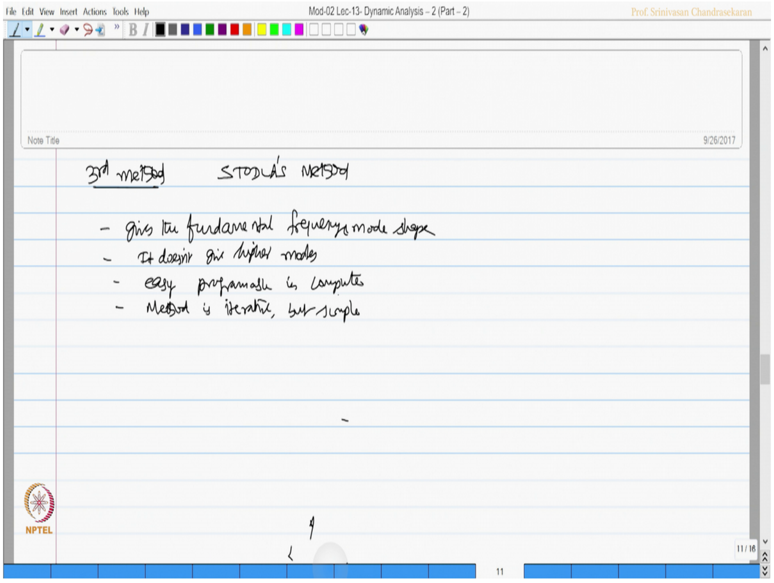

So, Stodola method gives the fundamental frequency and mode shape it does not give higher modes the method is easily programmable by computer though the method is iterative, but simple.

Let us take the same example and solve. So, the example 3, but similar to example 2 that is m 1 m 2 m 3, this is x 1 of t, this is x 2 of t, this is x 3 of t which is same as this example, same as this example provided ms and ks are well defined in the same fashion.

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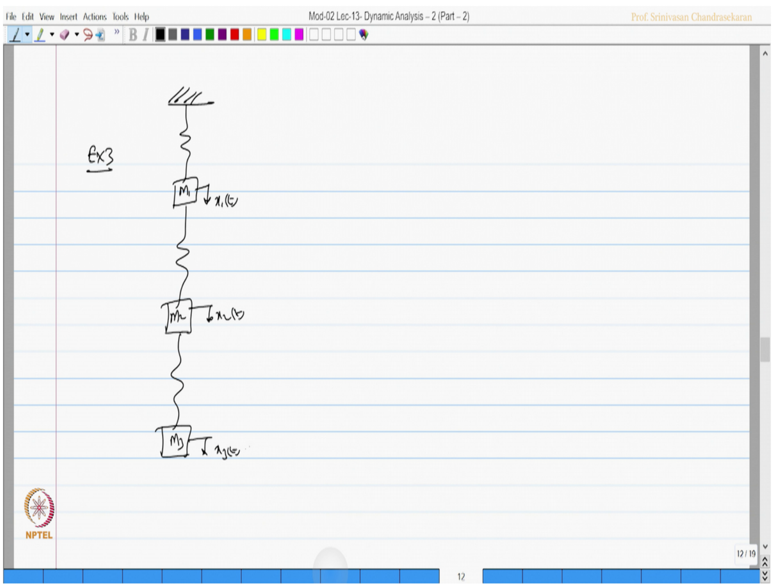

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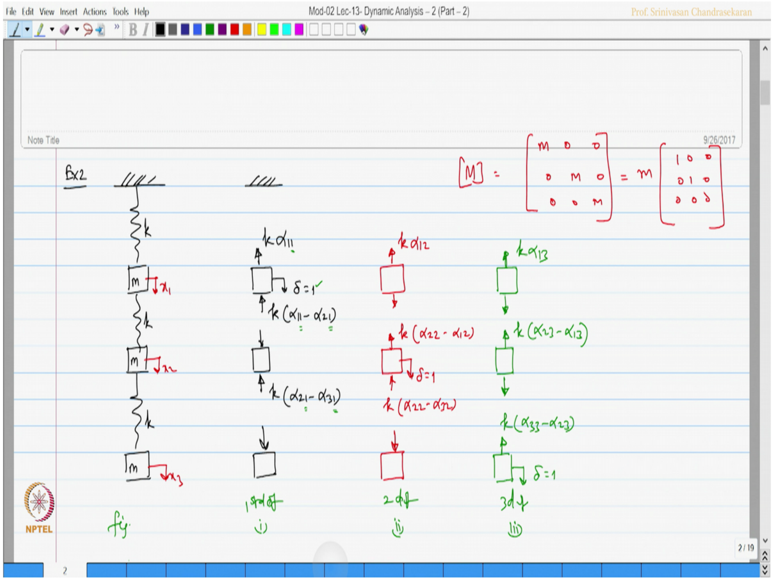

So, let m 1 be m 2 be m 3 be m, similarly this is k 1 this is k 2 this is k 3 let k 1 be k 2 be k 3 is k. So, let us try to solve this problem using Stodla's method. So, let us it is a tabular form let us say assumed deflection say k 1 m 1, k 2 m 2 and k 3 m 3. Let us say the assume deflection is at the mass points 1 1 and 1.

The inertia force could be m 1 omega square 1 and this is going to be m 2 omega square deflection, m 3 omega square deflection.

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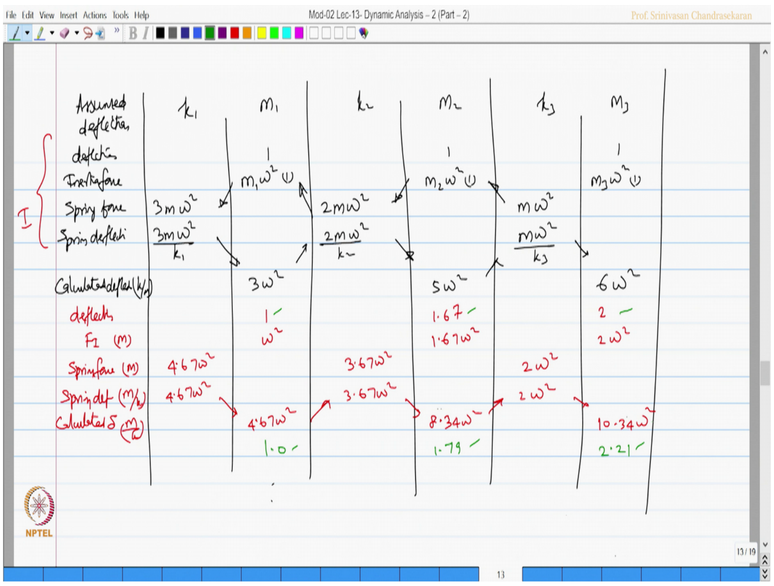

Now, I want to compute the spring force the spring force for k 3 that is the spring force for k 3 will be actually equal to m omega square and for k 2 it will be 2 m omega square it is because m 2 and m 1 n all equal to m, there is the reason. Similarly for k 1 it will be 3 m omega square I want to find the spring deflection, I divide this by k 1 so 3 m omega square by k 12, m omega square by k 2, m omega square by k 3.

So, now I want to compute the calculated deflection. So, I keep m and k out. So, I should say here this is going to be 3 omega square I should say k by m and phi omega square and 6 omega square. So, now, we can find the ratio keep this as 1, keep this as 1 so this will become 1 0.67 and this will become 2. So, I started with 1 1 1 I got 1, 1.67 and 2 so no convergence.

So, I will repeat the same case starting at 1, 1.67 and 2. So, I would say these are my deflections. So, this is the first iteration, deflection is 1, 1.67, 2 I calculate inertia force I take multiplier m out. So, this is going to be 2 omega square, there is 1 0.67 omega square this is omega square. I calculate the spring force again m out which is going to be 2 omega square, 3 0.67 omega square, 4 0.67 omega square. I say spring deflection again I say it is m by k 4.67 omega square, 3.67 omega square, 2 omega square. I get calculated deflection is m by k is going to be 4.67 omega square, 8.34 omega square just sum them up and 10.34 omega square. Now, I take a ratio take this as 1 this becomes 1.79 and this becomes 2.21. I started with 1, 1.67 and 2, but ended in 1; 1.79 and 2.21 again no convergence.

Let us do the third cycle assume deflection, which is let us say this is k 1 this is m 1, this is k 2 this is m 2, this is k 3 this is m 3.

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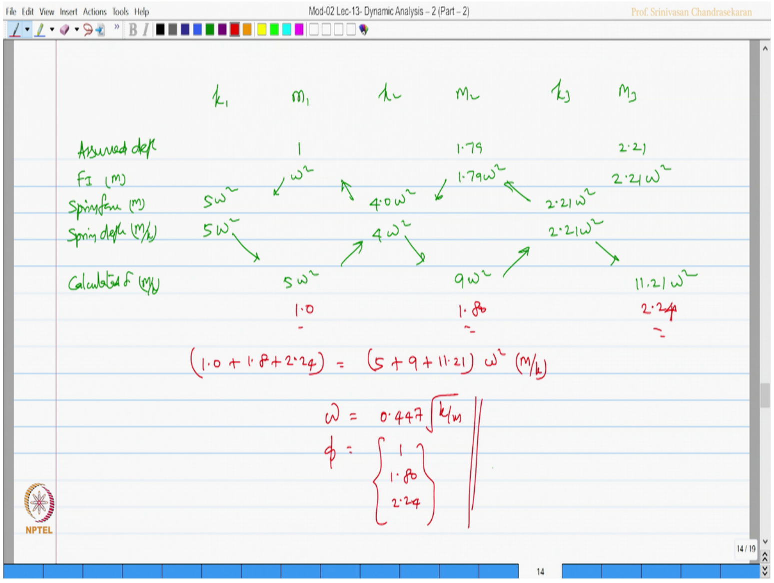

So, assume deflections are 1, 1.79 and 2.21 that is what we are borrowed from the last iteration. So, the inertia force m out will be 2.21 omega square 1.79 omega square and omega square spring force with m out will be 2.21 omega square which becomes 4.0 omega square which becomes 5 omega square as summation. Then I find spring deflection so divided by the stiffness, so it is going to be 5 omega square 4 omega square 2.21 omega square.

Now, I say calculator deflection I say this is going to be 5 omega square I get this at the mass points this is going to be 9 omega square and 11.21 omega square. I now take a ratio say this is 1, this is 1.80 and this is 2.24. I believe that it has converged because the previous iteration the values were 1, 1.79 and 2.21; now it is 1, 1.8 and 2.24. Once it is converged we say that 1 plus 1.8 plus 2.24 will be equal to 5 plus 9 plus 11.21 omega square m by k. So, if you are called I get omega as 0.447 root k by m.

So, if you look at the results of influence coefficient method we got 0.445 k by m and the mode shapes were 1, 1.801 and 2.25. Now we let us see what result do we get from here omega is 0.447 and the corresponding phi is 1, 1.80 and 2.24. Let us quickly compare these results 0.447 and this whereas this value is 445 and 1.8, 2.25. See very closely they are matching.

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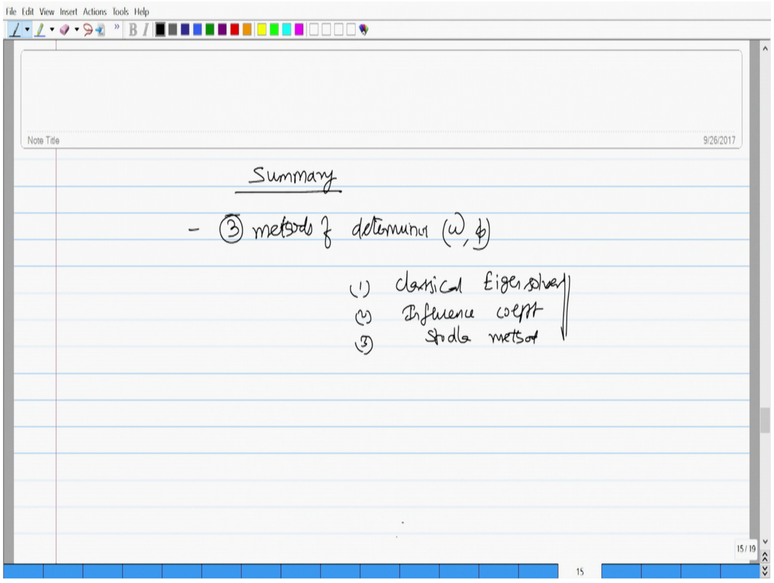

So, we learnt 3 methods in this lecture, we learned 3 methods of determining omega and phi that is a natural frequency and more shapes the methods are Classical Eigen solver method to Influence coefficient method, 3 Stodola method. All these are easily programmable we will be giving you also the computer methods of this and write the program and then solve the problem using the computer program and compare the results.

Thank you very much.