

Computer Methods of Analysis of Offshore Structures
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Module - 02
Lecture - 14
Dynamic Analysis - 3 (Part - 1)

So, friends let us continue the discussion on the 14th lecture in module 2.

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Module 2

Lecture 14: Dynamic Analysis - III

So far, computer methods of estimating (ω_n, ϕ_n) are discussed

- Eigen-solver
- Influence coefft
- Stodola

easily programmable

- solution is iterative, and needs an algorithm to solve

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We are discussing computer methods of estimating natural frequency and mode shapes. So far in computer methods of estimating natural frequency and mode shapes are discussed we have discussed Classical Eigen solver method, we have discussed Influence coefficient method, we have discussed **Stodola's** method, all are easily programmable. The solution is iterative and needs an algorithm to solve.

Let us now discuss one more method which is the 4th method which is given by Rayleigh which is again a numerical method which is easily programmable.

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(4) Rayleigh method

Mass (m)	deflection (ϕ_i)	$F_i = \alpha M_i \phi_i$ ($\omega^2 m$)	computed deflection $A' \phi_i$ ($\omega^2 m$)	mode shape ϕ_i	$F_i \phi_i$ ($\omega^2 m$)	$M_i \phi_i^2$ ($\omega^2 m$)
m	1	α	5α	1	α	1
2m	2	$\frac{4\alpha}{5}$	6.5α	2-b $(\frac{6.5}{2.5})$	10.4α	$= 2m(2-b)^2$ $= 13.52$
			$A' = 2.5\alpha$		$\sum 11.4\alpha$	$\sum 14.52$
		$\omega^2 = \frac{\sum F_i \phi_i^2}{A'^2 \sum M_i \phi_i^2} = \frac{11.4\alpha}{(2.5\alpha)^2 (14.52)} = \frac{11.4}{6.25 \times 14.52} = 0.56 \sqrt{k/m}$				$\omega = 0.56 \sqrt{k/m}$

Let us take an example, demonstrate this method let us take a problem which I am drawing here, it is a this is 2k, this is k, this is m and this is 2m and the degrees of freedom are this is x 1 this is x 2, we want to find omega and phi for this system.

So, let us take Rayleigh method it is again a tabular form numerical in method, let us say mass deflection which I call as phi r which I call as mr, inertia force which is alpha mr phi r where units of omega square m are kept outside. Then we say computed deflection which is A double prime of phi r then we say mode shape which is phi r double prime then we say F I of phi r double prime then we also find mr phi r double prime square.

Let us try to make a table enter these values. Let us say I want enter mass m and 2 m these are the mass values. I assumed the deflection with no zero crossing as 1 and 2 arbitrarily I assumed this value, I take the deflections as the spring values which is mr phi r which is alpha omega square m is constant. Here mr phi r which becomes 4 alpha omega square and mr taken as constant.

So, I make this sum the sum is phi alpha. I find this value is actually equal to this phi alpha divided by this stiffness which is I write here as 2.5 alpha and I will have a constant omega square m by k 2.5 alpha. By the same logic I can say this will be 4 alpha by k, but the summation will be 2.3 plus 4, 6.5 alpha.

Now, I take a multiplier a double prime as 2.5 alpha therefore, the mode shapes become 1 and 2.6, this 2.6 nothing but 6.5 by 2.5. Once they do this I will then find F I multiplied by phi r which is simply alpha and 4 alpha into 2.6 that gives me 10.4 alpha I sum this up I say 11.4 alpha. Then in the last column I find mr multiplied by phi r square. So, this is going to be simply 1, here also I have omega square m constant and here also I have omega square m constant and this is going to be equal to 2m multiplied by 2.6 square which is 13.52. So, I make the sum this is 14.52 and so on.

Now, I want to find omega. So, I get omega I get omega as sum of F I phi r that is this column divided by a double prime which is this value into sum of mr phi r square that is this column. So, that is going to be equal to 11.4 alpha by 2.5 alpha into 14.52.

So, we substitute the units you will see this value will be in omega square m this value will be omega square m they gets canceled, they will cancel and this will be having a multiplier omega square m by k. So, I get omega square m by k. So, k by m is subset I can say that, this omega square k by m.

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mass	deflection	F_I	Computed deflection	mode shape ϕ_j	$F_I \phi_j$	$m r \omega^2$
m	1	α	$\frac{6.2k}{3k} = 2.07k$	1	α	1
$2m$	2.6	5.2α	$\frac{8.3k}{3k} = 2.77k$	2.6	13.936α	14.936α
		6.2α	$A^0 = 3.1\alpha$		14.936α	15.365α
m	1	α	3.1α	1.0	α	1
$2m$	2.6	5.36α	8.54α	2.6	14.41α	14.472α
		6.36α	$A^0 = 3.18\alpha$		15.418α	15.472α

$\omega = 0.56 \sqrt{k/m}$
 $\phi = \begin{Bmatrix} 1 \\ 2.6 \end{Bmatrix}$

$\omega^2 = \frac{15.418\alpha}{3.18\alpha (15.472\alpha)} \sqrt{k/m} = 0.56 \sqrt{k/m}$

So, if we substitute that and take a root I get omega as 0.56 root k by m and the corresponding vector is 1, 2.6 that is this value. I started with 1 and 2 I got 1 and 2.6 non converging so one more iteration. So, again mass deflection inertia force, computed deflection, mode shape, inertia force multiplied by phi r then mr multiplied by phi r

square. Let us borrow those values form a problem this is m and this is $2m$ and the previous iteration it is 1 and 2.6, I will take that value as 1 and 2.6.

So, this going to be $\alpha = 5.2$ α I request you to follow the same algorithm what we expressed in the last cycle of iteration, last cycle of iteration the same way you get F_r you get F_I . Then this is going to be this value divided by α . So, let us take the sum is going to be 6.2α . So, this is going to be 6.2α by $2k$. So, that is the stiffness. So, $2k$ we just I think I should write 3.1α then you add them up it becomes 8.3α and the sum. So, I take a double prime as 3.1 , α I get the mode shape as 1 and this going to be equal to 8.3α by 3.1α which is 2.68 .

Then this is $F_I \phi$ into ϕ_r , so ϕ_r is this value this is ϕ_r^2 . So, it is going to be α this is 13.936α I make the sum which is 14.936α . So, this is $m r$ into ϕ_r square which is going to be 1 and this is going to be $2m$ multiplied by 2.68 square which is equal to 14.365 , I make the sum which is 15.365 . So, now, I find ω in the same algorithm I should say ϕ_I , $F_I \phi_I$ double prime divided by a double prime and this which is going to be equal to 14 point ω square, is 14.936α divided by 3.1α and 15.365 which will give me ω as $0.56 \text{ root } k \text{ by } m$.

And the corresponding vector is 1 2.68 . I do the next iteration I take this as m and $2m$ I take 1 and 2.68 , this is α this is 5.36α sum becomes 6.36α , so this becomes 3.18α , this is 8.54α and therefore, I take a prime as 3.18α I get more chip as 1 and 2.69 that is my motive 1 and 2.69 . Then you work on this value as α 14.418α I make the sum which is 15.418α this is 1 and this is 14.472 , I make the sum it is 15.472 . Eigen ω square as 15.418α by 3.18α multiplied by 15.472 which of k by m which gives me $0.56 \text{ root } k \text{ by } m$ as ω and ϕ is this value.

So, we will believe that it has converged and the final answer ω is $0.56 \text{ root } k \text{ by } m$ and ϕ is 1 and 2.69 since there is no zero crossing I can call this ω 1 and ϕ 1 respectively.