

**Computer Methods of Analysis of Offshore Structures**  
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**Module - 02**  
**Lecture - 14**  
**Dynamic analysis - 3**

Let us try to solve the same problem for experience by Stodla. So, let us say  $k_1$  and  $m_1$  let us say  $k_2$  and  $m_2$ ,  $k_1$  in the given problem you can see the problem  $k_1$  is  $2k$  and  $k_2$  is  $k$ ,  $m_1$  is  $m$  and  $m_2$  is  $2m$ . So, will use that  $k_1$  is  $2k$   $m_1$  is  $m$ ;  $k_2$  is  $k$  and this is  $2m$ .

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	$k_1$ ( $2k$ )	$m_1$ ( $m$ )	$k_2$ ( $k$ )	$m_2$ ( $2m$ )
Assumed $\delta$		$2.0$		$1.0$
$F_I$ (m)		$2\omega^2$		$2\omega^2$
Spring force (m)	$4\omega^2$		$2\omega^2$	
Spring $\delta$ (m/k)	$2\omega^2$		$2\omega^2$	
Calculated $\delta$ (m/k)		$2\omega^2$		$4\omega^2$
Assumed $\delta$		$1.0$		$2.0$
$F_I$ (m)		$\omega^2$		$4\omega^2$
Spring force (m)	$5\omega^2$		$4\omega^2$	
Spring $\delta$ (m/k)	$2.5\omega^2$		$4\omega^2$	
Calculated $\delta$ (m/k)		$2.5\omega^2$		$6.5\omega^2$
		$1.0$		$2.0$

Let us say assume deflection we will assume the deflection at the mass points let us call this as  $1.0$  and  $2.0$  inertia force will take  $m$  out. So, this is going to be  $2\omega^2$  this is also going to be  $2\omega^2$ . Then we say spring force again mass out, so this is going to be  $2\omega^2$  and  $4\omega^2$ . Let us find out the spring deflection. So, divide this by the stiffness of the spring this is going to be  $2\omega^2$  because  $4\omega^2$  by  $2k$  is  $2\omega^2$   $m$  by  $k$  and this is going to be also  $2\omega^2$ .

Now, let us say calculated deflection with the  $m$  by  $k$  out which is going to be  $2\omega^2$  and  $4\omega^2$  now the values are  $1.0$  and  $2.0$  there is no convergence will

take this as assume deflection. Then will compute the inertia force with mass out is going to be 4 omega square this is omega square, let us compute the spring force with mass out is going to be 4 omega square and 5 omega square. Then we say spring deflection we say k out, so 5 by 2 that is 2.5 omega square m by k, then this is 4 omega square m by k. Let us say calculated deflection with m by k out is going to be 2.5 omega square and 6.5 omega square let us find the ratio of 1 and 2.60. Again converging did not happen we will do one more cycle we will say k 1 m 1 k 2m 2, this is 2 k this is m, this is k this is 2m.

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	$k_1$ (2k)	$m_1$ (m)	$k_2$ (k)	$m_2$ (2m)	
Assumed $\delta$		1.0		2.60	
FI (m)		$\omega^2$		$5.2 \omega^2$	
Spring force (m)	$6.2 \omega^2$		$5.2 \omega^2$		
Spring deflection (m/k)	$3.1 \omega^2$		$5.2 \omega^2$		
Calculated $\delta$ (m/k)		$3.1 \omega^2$		$8.3 \omega^2$	
		1.0		2.67	
					$(1 + 2.67)$ $= (3.1 + 8.3) \omega^2 (m/k)$
					$\omega = 0.567 \sqrt{k/m}$
					$\begin{Bmatrix} 1 \\ 2.67 \end{Bmatrix} = \phi$

So, we have done this is the first cycle and this is the second cycle, now we are doing the third cycle of iteration we say assume deflection. So, we assume this value as the third iteration start value. So, the values are entered the mass point 1.0 and 2.6, we compute the inertia of force with mass constant, which is 5.2 omega square this is omega square we find the spring force which is 5.2 omega square 6.2 omega square then the spring deflection with m by k, so 6.2 by 2 that is 3.1 omega square m by k this is 5.2 omega square. So, we say calculate the deflection with m by k which is 3.1 omega square with 8.3 omega square we take the ratio this is 1.0 this is 8.3 by 3.1 which is 2.6.

We started with 1, 2.6; we landed in 1, 2.67. We assume that they are almost converged we say that 1 plus 2.67 is actually equal to 3.1 plus 8 point 3 omega square m by k is in under say constant out. So, now, I find omega square as 0.567 root k by m and the

corresponding phi the mode shape is 1 and 2.67. Let us compare this answer with what we got with Rayleigh method. Rayleigh method says omega is 0.56 and 1 2.6 and this method says 0.567, 1.267.

Let us solve this problem also by influence coefficient method. So, it is very simple I have to derive the influence coefficients from the first principles let us do that.

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**Influence Coefft method**

**1st dof**

$$k(\alpha_{11} - \alpha_{21}) = 0 \quad \alpha_{11} = \alpha_{21}$$

$$2k\alpha_{11} + k(\alpha_{11} - \alpha_{21}) = 1$$

$$\alpha_{11} = \frac{1}{2k} = \alpha_{21}$$

**2nd dof**

$$k(\alpha_{22} - \alpha_{12}) = 1$$

$$2k\alpha_{12} = k(\alpha_{22} - \alpha_{12})$$

$$= 1$$

$$\alpha_{12} = \frac{1}{2k}$$

**Stiffness Matrix M:**

$$M = \begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}$$

**Influence Coefficients:**

$$\alpha = \frac{1}{2k} \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

Additional calculations shown:

$$\alpha_{22} - \alpha_{12} = \frac{1}{k}$$

$$\alpha_{12} = \frac{1}{k} + \alpha_{12} = \frac{1}{k} + \frac{1}{2k} = \frac{3}{2k}$$

This is going to be m 2m this is 2 k and k, let us write the free body diagram so unit force. So, this is going to be 2 k of alpha 11 and this is going to be k of alpha 11 minus 21. Similarly for the second degree give unit force here. So, this is going to be k times of alpha 22 minus alpha 12 and this is going to be 2 k of alpha 12.

So, let us write down for the first degree we say this is unity this unity. So, k of alpha 11 minus 21 is equal to 0 which implies that alpha 11 is alpha 21. 2 k of alpha 11 plus k of alpha 11 minus 2 1 is unity. Substituting that this value is 0 you know alpha 11 will become 1 by 2 k which is also give me as alpha 21.

Now, for the second degree k times of alpha 22 minus 12 is unity, then 2 k of alpha 12 is k of alpha 22 minus 12. So, k of alpha 22 minus 12 is equal to unity so I can say this is 1 which implies that alpha 12 is 1 by 2 k. Now we know alpha 22 minus alpha 12 is 1 by k. So, alpha 22 is 1 by k plus alpha 12 which is 1 by k plus 1 by 2 k which I get this as 13 by 2 k.

So, now I have the alpha matrix with 1 by 2 k constant out. So, this going to be alpha 11 and 21 is 1 and 1, alpha 12 is again 1 and this is 3. And for this problem the mass matrix is m 0 0 2m. Now I write the control equation for iteration we know it is going to be x 1 x 2 is actually omega square m by 2 because m and 2 k are constants in alpha in m matrices. So, m by 2 k then I have to multiply alpha matrix with m. So, I get the first column as simply 1 that is this and this, so 1 and this with this is going to be again 2 is going to be 1 and 6 of x 1 x 2 because this is m and 2m therefore, this is 1 2 and 1 and 6.

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$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{\omega^2 m}{2k} \begin{bmatrix} 1 & 2 \\ 1 & 6 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{\omega^2 m}{2k} \begin{bmatrix} 1 & 2 \\ 1 & 6 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{\omega^2 m}{2k} (5) \begin{pmatrix} 1 \\ 2.6 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2.6 \end{pmatrix} = \frac{\omega^2 m}{2k} \begin{bmatrix} 1 & 2 \\ 1 & 6 \end{bmatrix} \begin{pmatrix} 1 \\ 2.6 \end{pmatrix} = \frac{\omega^2 m}{2k} (6.2) \begin{pmatrix} 1 \\ 2.68 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2.68 \end{pmatrix} = \frac{\omega^2 m}{2k} \begin{bmatrix} 1 & 2 \\ 1 & 6 \end{bmatrix} \begin{pmatrix} 1 \\ 2.68 \end{pmatrix} = \frac{\omega^2 m}{2k} (6.36) \begin{pmatrix} 1 \\ 2.69 \end{pmatrix}$$

Let us assume the iteration as 1 and 2 which is omega square m by 2 k of 1 2 1 6 of 1 and 2. So, this has a multiplier omega square m by 2 k of 5 of 1 and 2.6. So, the second iteration 1 2.6 omega square m by 2 k - 1 2 1 6 of 1 and 2.6 which is omega square m by 2 k of 6.2 of 1 2.68.

So, we take the next iteration 1 2.68 omega square m by 2 k, 1 2 1 6 of 1 2.68 this gives me omega square m by 2 k the multiplier of 6.36 and I get the vector 1 2.69. I do one more iteration.

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$$\begin{Bmatrix} 1 \\ 2.69 \end{Bmatrix} = \frac{\omega_m^2}{2k} \begin{bmatrix} 1 & 1 \\ 1 & 6 \end{bmatrix} \begin{Bmatrix} 1 \\ 2.69 \end{Bmatrix}$$

$$= \frac{\omega_m^2}{2k} (6.38) \begin{Bmatrix} 1 \\ 2.69 \end{Bmatrix}$$

$$1 = \frac{\omega_m^2}{2k} (6.38) (1); \quad \omega_1 = 0.56 \sqrt{k/m}$$

$$\phi_1 = \begin{Bmatrix} 1 \\ 2.69 \end{Bmatrix}$$

1 2.69 omega square m by 2 k, 1 2 1 6 of 1 2.69 which gives me omega square m by 2 k multiplier of 6.38 of 1 2.69, I get the same conversion value therefore, 1 will be equal to omega square m by 2 k of 6.38 of 1 which implies that n omega is equal to 0.56 root k by m and the corresponding 5 is 1 2.69 no zero crossing therefore, first frequency and first mode shape. Let us compare this answer with what I got from Stodla, so 0.567 and 1 2 6 7, so 0.567, 1, 2.6. So, the answers are completely matching.

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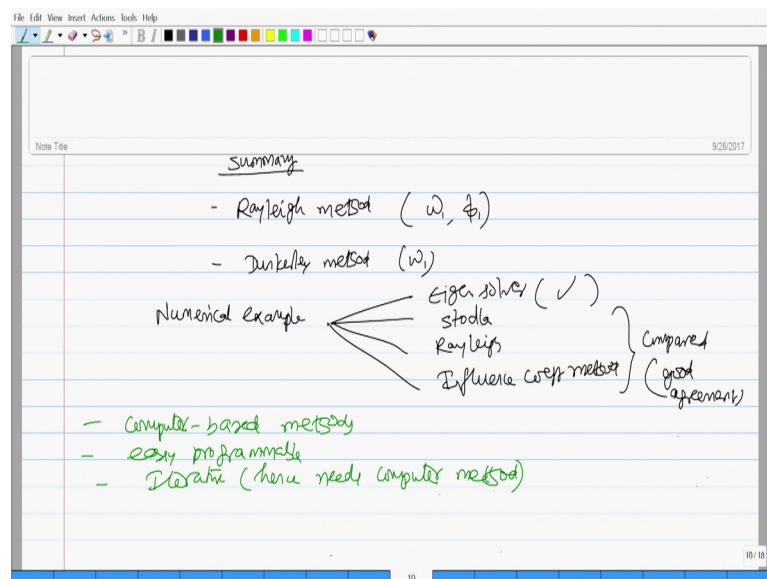
Dunkerlay (approximate) -  $\omega_n$  (not the mode shape)

$$\frac{1}{\omega_n^2} = \sum m_i d_i^2$$

$$\frac{1}{\omega_n^2} = m \left(\frac{1}{2k}\right) + (2m) \left(\frac{3}{2k}\right)$$

One can also verify this with another method which is given by Dunkerlay which say an approximate method it can also be used to compute only the fundamental frequency and not the mode shape. So, this says  $1 \text{ by } \omega_1^2$  there is a fundamental frequency is some of  $m_1 \alpha$  (Refer Time: 14:38) i. So, if you would not really find out for this problem it is going to be  $1 \text{ by } \omega_1^2$  which will be equal to  $m_1$  is  $m$  and  $\alpha$   $11$  is  $1 \text{ by } 2k$  plus  $m_2$  is  $2m$  and this is  $3 \text{ by } 2k$  this will give you approximate to the value of  $\omega$ . I leave it to you to compute this, but does not give you  $\phi$ .

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So, as summary friends in this lecture we learned Rayleigh method of estimating fundamental frequency and mode shape. We also learnt an approximate Dunkerley method of estimating only the fundamental frequency. We also solved a numerical example by Eigen solver the of course, we did not do it here in this class, but I hope you must have known to do it, but also solved it with Stodola, with Rayleigh, with influence coefficient method and we compared the answers and they were found to be in good agreement. So, the point is all these methods are computer based methods they are easily programmable and they are iterative and hence needs computer method.

So, in the next lecture we would like to give you the coding of these, solve a problem and then compare the results using computer code which are exactly same as that of the results of what you got behind.

Thank you very much.