

Computer Methods of Analysis of Offshore Structures
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Module - 02
Lecture - 16
Damping Estimate (Part – 1)

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Module 2
Lecture 16: Damping Estimate I

(ω , b) — Numerical methods

- computer programs are available
- sample problems - code
- compared the results for validity

standard input $[M]$, $[k]$, d-o-f

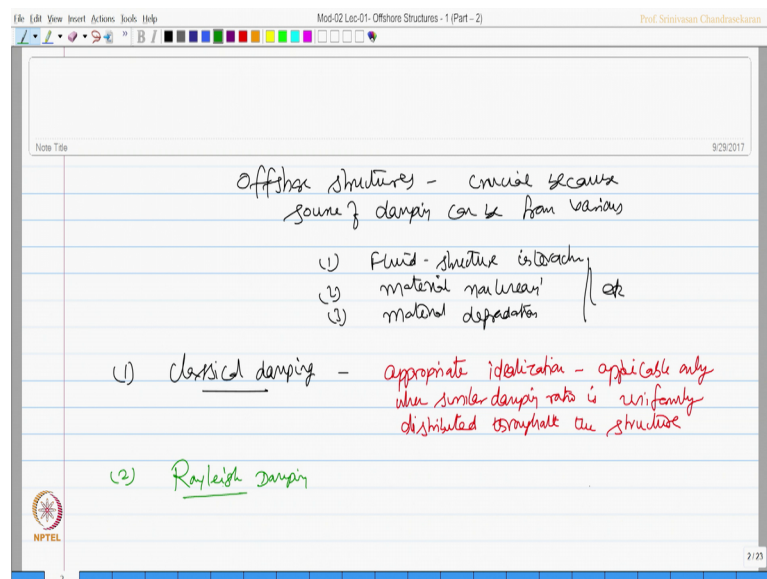
natural frequencies (ω_n) — Eigenvalues
mode shapes (ϕ_n) — Eigen vectors

— Damping Estimate is one of the important parameter to be evaluated.

So, friends let us welcome to the 16th lecture in module 2 where we are going to talk about damping estimates. As a quick revamp we already said one can estimate natural frequency and mode shape using various numerical methods for which the computer programs has been already given. We have also solved some sample problems using the computer code and compare the results for validation. You can recollect very well that in these methods we **have** standard inputs as follows, you must input the mass matrix, you must input the stiffness matrix, you have to also state the number of degrees of freedom based on which you will be able to get the natural frequencies ω_n and the corresponding mode shapes ϕ_n , otherwise referred as eigen values and eigen vectors.

In dynamic analysis we realized that damping estimate is one of the important parameter to be evaluated.

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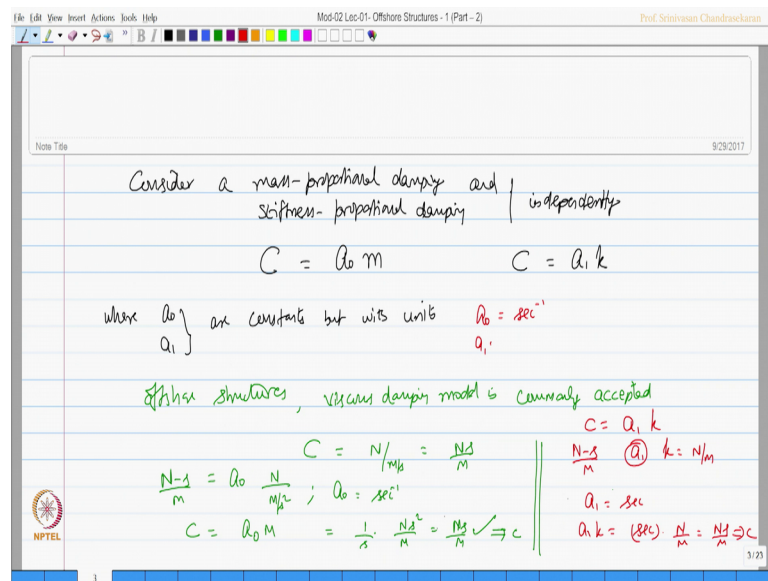
There are many methods by which damping can be estimated, but in offshore structures damping estimates become very crucial because source of damping can be from various sources. For example, it can be arising from fluid structure interaction, it can also arise from material non-linearity, it can also arise from material degradation etcetera.

So, let us see how damping estimates can be made for the known mass and stiffness matrices. When you talk about damping a very common class of reference in engineering literature is classical damping. Classical damping is actually an appropriate idealization

and is applicable only when similar damping ratio is uniformly distributed throughout the structure.

Alternatively people have recommended a new kind of damping in the literature which is proposed by Rayleigh and named after Rayleigh damping. Let us see what Rayleigh damping insist upon.

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Let us consider a mass proportional damping and stiffness proportional damping independently that is C is a $a_0 m$ that is mass proportional damping or C is a $a_1 k$ where a_0 and a_1 are constants, but with units. We strongly agree and believe that in offshore structures viscous damping model is commonly accepted. In viscous damping model we know the unit for damping could be Newton per meter per second which is Newton second per meter.

So, keeping that as a unit Newton second per meter should be a a_0 of Newton per meter per second square. So, therefore, a a_0 unit is coming to be per second. We can substitute and see C should be a $a_0 m$ which means per second Newton per meter per second square I get Newton second per meter which is as same as C .

Similarly, C is also equal to a $a_1 k$ this is Newton second per meter and a a_1 we do not know the units k is Newton per meter so; obviously, unit of a_1 is second. So, a $a_1 k$ will be second multiplied by Newton per meter which is Newton second per meter which is

as same as unit of C. So, now, we have constants a 0 with unit per second and a 1 with unit second.

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In both the damping cases, i.e. Mass-proportional & stiffness proportional, C is diagonal (by virtue)

- due to modal orthogonality
- Classical damping matrices
- physically, they can be represented as below

In mass-proportional damping, damping caused by air/atmosphere can be negligible - but in offshore structures, this can be significantly high

In both the damping cases that is mass proportional and stiffness proportional damping, C is diagonal by virtue this is due to modal orthogonality, these are called classical damping matrices.

Physically they can be represented as below. Let us take a multi story building with n number of floors each floor we have a mass lumped, let us say m_1, m_2, m_n and each mass will have a separate degree of freedom which are independent displacements. Now I can represent this as your damping model which is represented as here dash port model which I call this as a_{0m1}, a_{0m2} by that logic this is a_{0mn} this is mass proportional damping. In mass proportional damping, damping caused by presence of air or atmosphere can be negligible. But in offshore structures this can be significantly high.

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The diagram shows a three-story building with two columns. Each column has a stiffness k at each floor level. The damping is represented by red diagonal lines connecting the mass points of adjacent floors. The damping coefficient for each floor is labeled as $a_k k$, where a_k is a constant and k is the stiffness of the column. The building is supported by fixed bases. The text next to the diagram states: "dissipation of energy depends on relative displacement between the successive floors - the successive mass p".

The next case could be a stiffness proportional damping. So, let us say we have again a multi story building we have each floor levels, each floor level will undergo relative displacement and now the damping model is expressed as a relative displacement between the floors.

So, this is going to be a $1 \times k \times 1$ this is a $2 \times k \times 2$ similarly this can be an $n \times n$ whereas, the stiffnesses are k_1, k_2, k_n respectively. In this case dissipation of energy depends on relative displacement between the successive floors to be very precise I should say it is between the successive mass points.

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(i) Keeping $[C]$ proportional to modal damping ratio,
 for a mass-proportional damping system, we get

$$C_n = \alpha_0 M_n \quad \text{--- (1)}$$

damping ratio will be $\xi_n = \frac{C_n}{2M_n\omega_n} = \frac{\alpha_0 M_n}{2M_n\omega_n} \quad \text{--- (2)}$

$$\xi_n = \frac{\alpha_0}{2} \frac{1}{\omega_n} \quad \text{--- (3)}$$

Damping ratio (ξ_n) is inversely proportional to the natural frequency of the system.

Therefore, α_0 can be selected to obtain a specified value of damping ratio in any mode of designer's choice.

Having said this let us say keeping C proportional to modal damping ratio for a mass proportional damping system we get C_n as $\alpha_0 m_n$, then the damping ratio will be $\xi_n = \frac{C_n}{2m_n\omega_n}$ that is a standard relationship which is substituted as $\frac{\alpha_0 m_n}{2m_n\omega_n}$ which I get $\xi_n = \frac{\alpha_0}{2} \frac{1}{\omega_n}$ which indicates that the damping ratio ξ_n is inversely proportional to the natural frequency of the system. Therefore, α_0 can be selected to obtain a specified value of ξ_n in any mode of your choice that is ξ_n can be designer's choice and α_0 will be $2\xi_n\omega_n$.

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$$\xi_i = \alpha_0 \omega_i \text{ designer's choice}$$

$$\alpha_0 = 2\xi_i\omega_i$$

once α_0 is determined, (ξ_i, ω_i) are chosen
 damping matrix C can be computed

$$C = \alpha_0 M \quad (\text{for known } \alpha_0 \text{ values})$$

Once a ω_i is determined it is a known value now because ζ_i and the corresponding ω_i are chosen damping matrix C can be computed which is C equals a θ of m for known ω_i values.

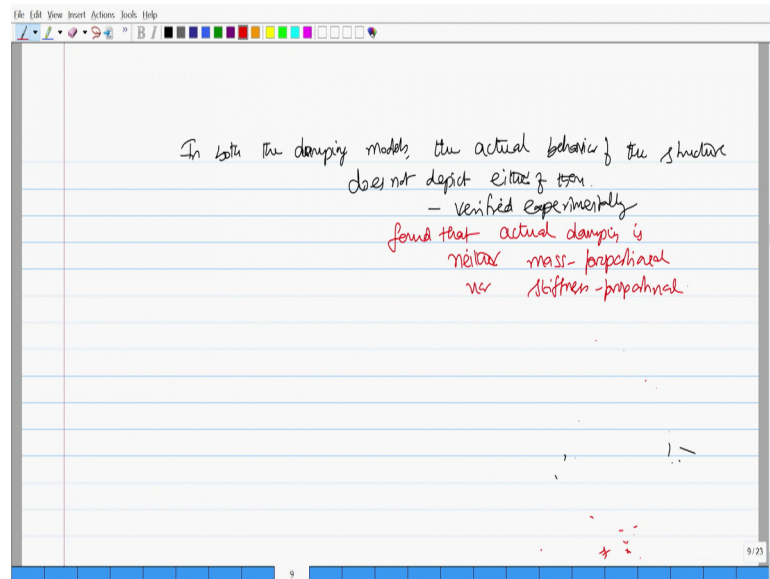
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For stiffness-proportional damping,
 $C_n = a_1 k$
 $C_n = a_1 \omega_n^2 M_n$ — (4)
 $\zeta_n = \frac{C_n}{2 M_n \omega_n} = \frac{a_1 \omega_n^2 M_n}{2 M_n \omega_n}$ — (5)
 $\zeta_n = \frac{a_1}{2} (\omega_n)$ — (6)
 Damping ratio (ζ_n) is directly proportional to natural frequency (ω_n)
 a_1 can be determined for (ω_i, ζ_i) for any mode, then $C = a_1 k$
 $a_1 = \frac{2 \zeta_i \omega_i}{\omega_i^2}$ — (7)

Similarly, for stiffness proportional damping C_n equals a $1 k$, C_n is a $1 \omega_n^2 m$ and ζ_n is equation number 4 $\zeta_n = C_n / 2 m \omega_n$ which is a $1 / 2 m \omega_n$. This says that the damping ratio ζ_n is directly proportional to natural frequency ω_n .

So, let us call this equation as phi and this equation as 6. a_1 can be determined for known value as ω_i and ζ_i for anymore then C simply is a $1 k$ and a 1 or a 1 is equal to $2 \zeta_j \omega_j$. So, one can find I call this is a_1 , a 1 and a 0 both constants, once you know these constants you can either find damping matrix as mass proportional or as stiffness proportional.

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But in reality there is a difference or there is a problem by doing these models. In both the damping models that is stiffness proportional and mass proportional the actual behavior of the structure depict either of them. This has been verified experimentally and found that actual damping is neither mass proportional nor stiffness proportional then what is the designer's choice.