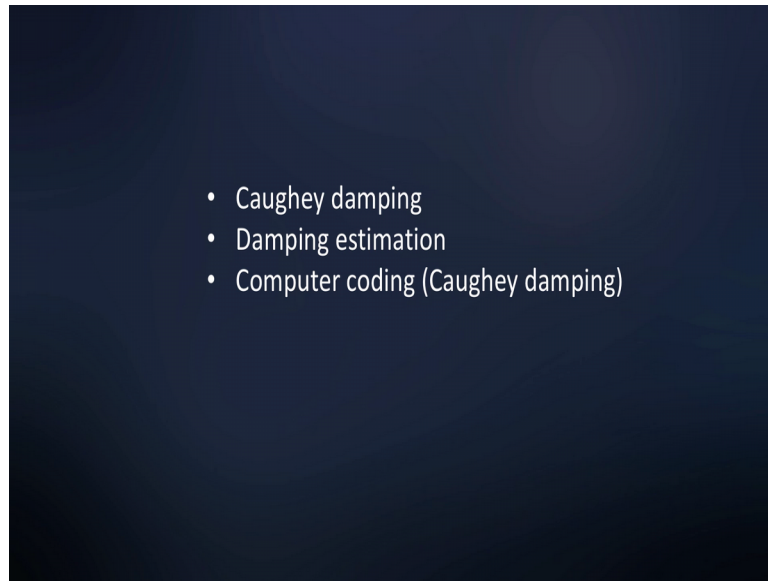


Computer Methods of Analysis of Offshore Structures
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Module - 02
Lecture - 17
Damping Estimate - 2 (Part - 2)

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File Edit View Insert Actions Tools Help Mod-02 Lec-17- Damping Estimate - 2 (Part - 2) Prof. Srinivasan Chandrasekaran

Note Title 9/29/2017

$$= \alpha_2 \omega_n^2 \underbrace{\phi_n^T k \phi_n}_{\omega_n^2 M_n \text{ for derivation}}$$
$$= \alpha_2 \omega_n^2 \omega_n^2 M_n$$
$$= \alpha_2 \omega_n^4 M_n$$

Substituting this above, in $G(u)$, we get

$$C_n = \sum_{l=1}^{N-1} \alpha_l \omega_n^{2l} M_n \quad (11)$$

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Which can be said as a $2 \omega_n^2 \phi_n^T k \phi_n$ can be actually said as $\omega_n^2 m_n$ by standard derivation; hence, a $2 \omega_n^2 m_n$ which is a $2 \omega_n^2 m_n$.

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for n^{th} mode, generalized damping is given by

$$C_n = \phi_n^T C \phi_n = \sum_{l=0}^{n-1} \phi_n^T c_l \phi_n \quad (10)$$

where $c_l = m \omega_n^l k^l$ — same as Eq (5) ✓

for $l=0$: $\phi_n^T c_0 \phi_n = \phi_n^T (a_0 m) \phi_n = a_0 M_n$

for $l=1$: $\phi_n^T c_1 \phi_n = \phi_n^T (a_1 k) \phi_n = a_1 \omega_n^2 M_n$

for $l=2$: $\phi_n^T c_2 \phi_n = \phi_n^T (a_2 k^2) \phi_n = a_2 \omega_n^4 M_n$

So, substituting for l equal to 0 1 and 2 in equation 10: substituting the above in equation 10 we get C_n will be summation of l equal 0, n minus 1. We can see the equation 10 $C_n = \sum_{l=0}^{n-1} \phi_n^T c_l \phi_n$. So, we have worked out $\phi_n^T c_0 c_1 c_2$ that will be the summation, right. So, it is going to be a l of $m_n \omega_n^{2l}$ equation 11.

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Damping ratio $\zeta_n = \frac{C_n}{2 M_n \omega_n}$

$$\zeta_n = \frac{1}{2} \sum_{l=0}^{n-1} a_l \omega_n^{2l-1} \quad (12)$$

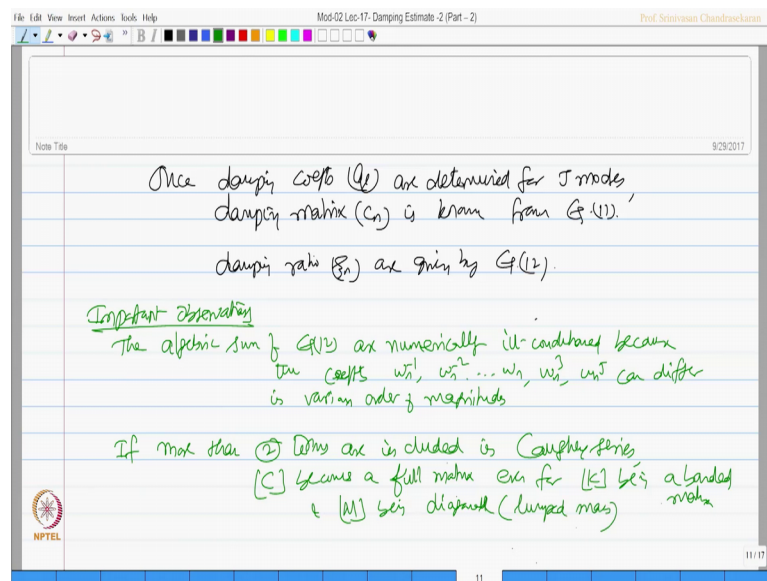
Coeffs of C , i.e. a_l can be determined from damping ratio spectrum for any J modes. This can be done by solving J algebraic eqns of (12).

for unknowns of a_l for $l=0, 1, \dots, J-1$, this above can be done.

Now damping ratio ζ_n is given by $C_n / 2 \omega_n$. So, ζ_n is now going to be half of summation of l equal to 0 to $N - 1$, $a_l \omega_n^{2l - 1}$ equation 12. So, it is interesting to know that the coefficients of damping matrix that is a l can be determined from various damping ratios specify for any J modes.

This can be done by solving J algebraic equations of equation 12. So, for unknowns of a l for l equal 0 1 etcetera till $J - 1$ this can be done the above can be done I do not think there is any confusion in this.

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Once the damping coefficients a_l are determined for J modes, then the damping matrix C_n is known from equation 11. And damping ratios ζ_n are given by equation 12; there are some important observations of Caughey series, the algebraic sum of the equation that is equation 12, algebraic sum of equation 12 are numerically ill conditioned because the coefficients ω_n^{-1} , ω_n^{-2} etcetera ω_n , ω_n^3 , ω_n^5 can differ in various order of n magnitudes.

Now, how to address this issue? If more than 2 terms are included in Caughey series, C becomes a full matrix even for K being a banded matrix and m being diagonal because masses damped.

So, what is the consequence of this?

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- This increases the computational cost - large systems

Ex

$m_1 = m_2 = m_3 = M = 3500 \text{ kg}$
 $k_1 = k = 1500 \text{ kN/m}$
 Assume $\zeta = 5\%$ in all modes
 Evaluate classical damping using Caughey series

$\omega_1 = 11.8 \text{ rad/s}$
 $\omega_2 = 29.27 \text{ rad/s}$
 $\omega_3 = 44.778 \text{ rad/s}$

$\phi_1 = \begin{Bmatrix} 1 \\ 0.68 \\ 0.32 \end{Bmatrix}$
 $\phi_2 = \begin{Bmatrix} 1 \\ -1 \\ -1 \end{Bmatrix}$
 $\phi_3 = \begin{Bmatrix} 1 \\ -3.68 \\ 4.68 \end{Bmatrix}$

The consequence is this increases the computational cost and time where then matrix now becomes a full matrix, especially when applied to large systems.

Let us take an example problem the same case what we did, let say this is m_1 m_2 m_3 ; m_1 m_2 and m_3 are M let us say it is 2500 kg. This is k_1 , this is $1.5 k_1$, this is $2 k_1$ and c_1 and c_2 is simply c which is 1500 kilo Newton per meter and let us say assume zeta 5 percent in all modes. So, using Caughey series let us evaluate. So, evaluate classical damping using Caughey series, that is the question asked.

We all know that ω_1 for this problem is 11.8 radians per second, ω_2 is 29.27 and ω_3 is 44.778 and the corresponding mode shapes also were known to us ϕ_1 is 1.68 and 0.32 ϕ_2 is 1 minus 1 and minus 1, and ϕ_3 is 1 minus 3.68 and plus 4.68 that is we already know.

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Handwritten notes on a digital whiteboard showing the derivation of the coefficient matrix A for the equation $2 \sum_n = \frac{a_0 + a_1 \omega_n + a_2 \omega_n^3}{\omega_n}$. The matrix A is defined as:

$$2 \begin{bmatrix} 0.05 \\ 0.05 \\ 0.05 \end{bmatrix} = \begin{bmatrix} 1/11.8 & 11.8 & (11.8)^3 \\ 1/29.27 & 29.27 & (29.27)^3 \\ 1/44.778 & 44.778 & (44.778)^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \quad (1)$$

find $A^{-1} \rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$, (a_0, a_1, a_2) find C as given by Caughey series
 $C = a_0 M + a_1 k + a_2 \frac{k^2}{m}$

So, we know that $2 \zeta_n$ is equal to a_0 by ω_n plus $a_1 \omega_n$, plus $a_2 \omega_n^3$ this is actually the value what we have from this equation ok.

So, $2 \zeta_n$ is we are taking three series 0 1 and 2; so a_0 , a_1 and a_2 . So, if it is 0, a_1 is 0 this becomes one by ω_n that is the term what we are getting here similarly if this is one. So, $\zeta_1 = a_1$ this becomes ω_1 that is what I getting here. The second term and similarly if this is 2; so 1 by 2^2 and this is going to be ω_n^3 that is what we are getting here. So, 3 terms, ok.

So, very interesting to I am assuming damping ratio same in all the modes classical damping which is expressed as in matrix form 1 by 11.8 that is my ω_1 , 1 by 29.27 ω_2 , 1 by 44.778 then ω_1 , ω_2 and ω_3 , then ω_1^3 , ω_2^3 and ω_3^3 of a_0 , a_1 , a_2 I can convert the above equation into a matrix form as given below.

Now, I call this matrix as A find A^{-1} . So, that will give you the coefficient matrix a_0 , a_1 and a_2 once you know a_0 , a_1 and a_2 , one can find C as given by Caughey series. So, which actually equal to $a_0 M$ plus $a_1 k$ plus $a_2 \frac{k^2}{m}$ inverse k this can be easily solved and found.

Let us take a computer program of this.

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2. CAUGHEY DAMPING
% Program for finding damping matrix using Caughey method
clear;
% Enter dof
dof=3;
%Enter Mass Matrix
m=[3500 0 0; 0 3500 0; 0 0 3500]; % mass in kg
%Enter Stiffness Matrix
k=[1500000 -1500000 0; -1500000 3750000 -2250000; 0 -2250000 5250000]; % stiffness in N/m
%Enter assumed damping ratio
drg=5.5%; %damping ratio in percentage
% damping ratio in percentage
dz=drg./100;
fprintf('Mass Matrix\n');
disp(M);
fprintf('Stiffness Matrix\n');
disp(K);
% eigen values and eigen vectors
[MODE,W_SQUARE]=eig(K,M);
freq=sqrt(W_SQUARE);
for i=1:dof
    wh(i)=freq(i);
    MODE(:,i)=MODE(:,i)/MODE(i,i);
    fprintf('frequency: wn = %6.3f rad/s \n',wh(i));
end
fprintf('Modal Matrix\n');
disp(MODE);
%% Caughey Damping
wn=sort(freq,'asc');
for i=1:dof
    wnwn(i)=wn(i)*wn(i);
end
a=2*(1/(wnwn))'; %constants
C=zeros(dof,dof); % damping matrix
for i=1:dof
    a=(a+((1/(wnwn(i))*K')*(K-wnwn(i)*M)'));
end
disp(C);

```

Output:

Mass Matrix	3500	0	0
	0	3500	0
	0	0	3500

Stiffness Matrix	1500000	-1500000	0
	-1500000	3750000	-2250000
	0	-2250000	5250000

Frequency: wn	11.721 rad/s
Frequency: wn	29.277 rad/s
Frequency: wn	44.783 rad/s

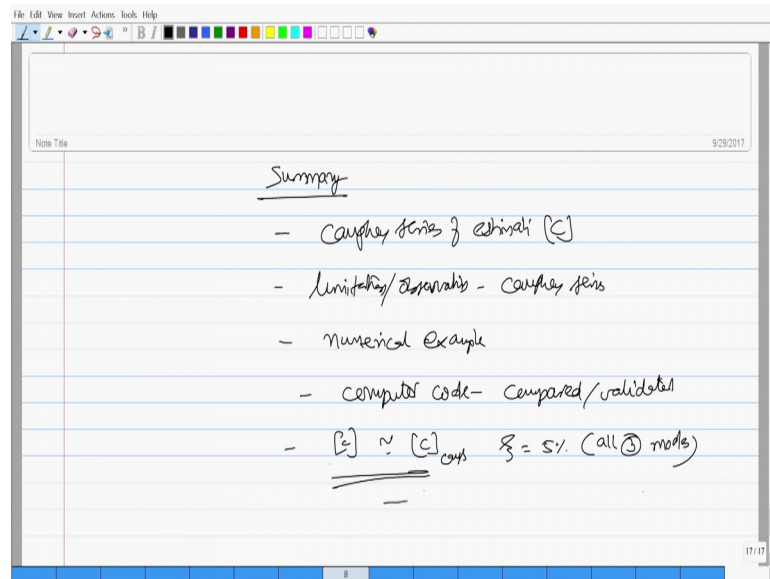
Modal Matrix	x
	1.0000 1.0000 1.0000
	0.6794 -1.0000 -3.6794
	0.3206 -1.0000 4.6794

Damping Matrix	1.0e+04 *
	0.6668 -0.3217 -0.0562
	-0.3217 1.0490 -0.3420
	-0.0562 -0.3420 1.3105

So, this is the program which is showing you the Caughey damping. We have to enter the degrees of freedom and enter the degrees of freedom give the mass matrix, give the stiffness matrix, assume damping same in all modes classical damping then of course, eigen values and eigen vectors are determined we already have the answers with us, get the modal matrix then find out the damping constants and then find the c matrix the program is actually continuing from here and to here. So, it is continuing here and then print. So, mass matrix is input stiffness frequencies corresponding mode shape and the damping matrix.

You will notice that the damping matrix obtained for all the three degrees including mode 1 2 and 3 is marginally different from what you obtain from Rayleigh damping in the last lecture.

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So, friends let us quick at the summary of this lecture, in this lecture we have learned the Caughey series of estimating classical damping matrix, we have also seen the limitations or observations of using the Caughey series. We have done a numerical example we have also **given** the computer coding to solve the problem and we compare the answers and validated.

You have also seen a damping matrix obtained by Rayleigh and damping matrix obtained by Caughey or more or less similar for zeta equals 5 percent in all 3 modes. I hope you have followed this lecture. You will attempt to solve this problem completely using a computer program, compare the answers for other application problems and let me know if you have any difficulties.

Thank you very much.