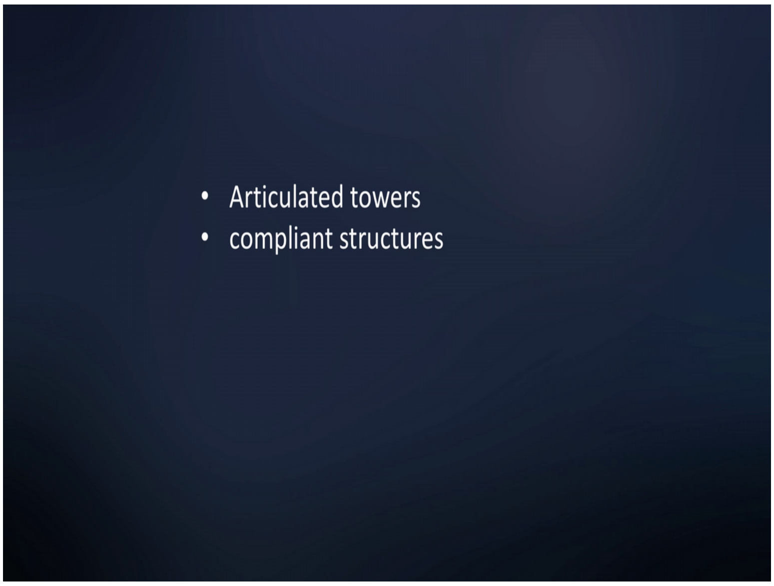
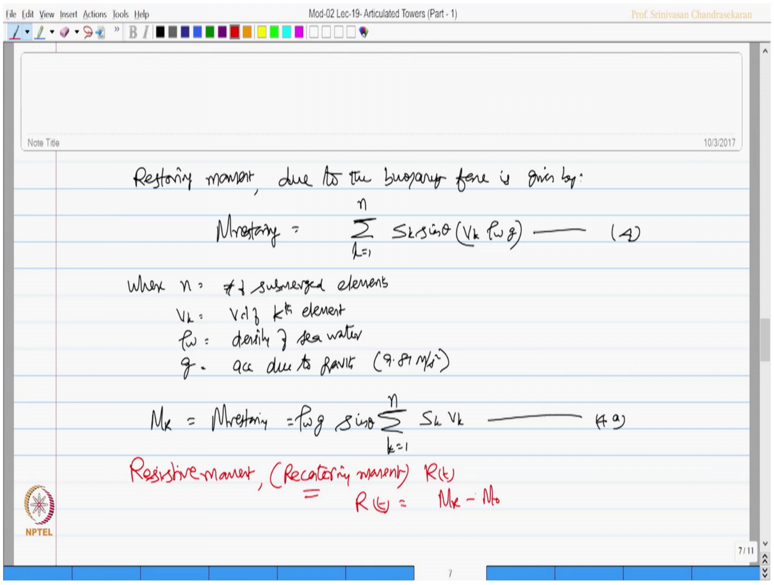
Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 02
Lecture - 19
Articulated Towers (Part - 2)

(Refer Slide Time: 00:16)



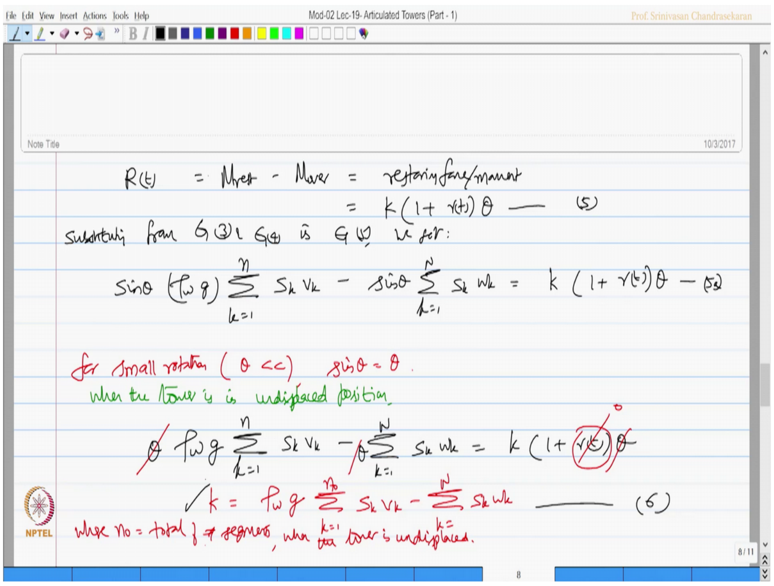
(Refer Slide Time: 00:24)



Now, the restoring moment which comes due to the buoyancy force is given by M restoring is again S k sin theta into V k into rho w g; sum this up for k equals 1 to n where n is the number of submerged elements and V k is the volume of k eth element, rho w density of sea water and g is acceleration due to gravity, which is 9.81 meter per second square. So, I call this as equation number 4. In fact, I rewrite equation number 4 straight in a different form as restoring moment is rho w g sin theta summation k equals 1 to n, S k V k, I call this as equation 4.

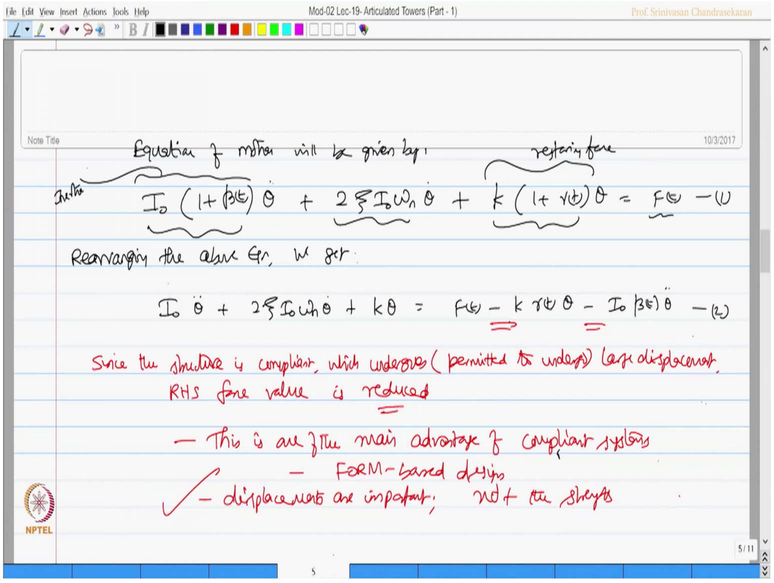
Now I have the (Refer Time: 02:09) turning moment which is M overturning, I have the restoring moment which is M R, now I can always find the resistive moment which is otherwise called as recentering moment. If you recollect it is important to note that compliant structures are generally designed to have a very high recentering capability therefore, let us try to find out the recentering moment which is given as R t, which will be actually equal to the restoring moment minus overturning moment.

(Refer Slide Time: 02:54)



The restoring moment minus overturning moment if I call this as R t from a original equation of motion, please look at the original equation of motion from the original equation of motion this is my restoring force.

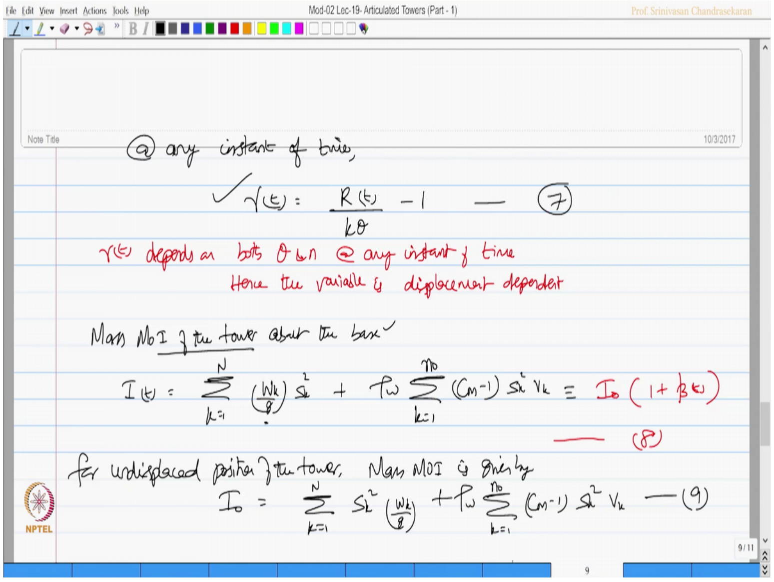
(Refer Slide Time: 03:08)



So, this should be equal to be restoring force in fact, restoring moment because I am taking moment above the hinge in all the cases. So, I should say this should be actually equal to k of 1 plus r of t into theta. So, now, I substituting, this is going to be equation 5 now substituting from equation 3 and equation 4 in equation 5, we get sin theta rho w g summation k equals 1 to n of Sk V k minus sin theta summation k equals 1 to N S k w k is actually equal to k 1 plus r t of theta. So, I call now this as equation 5 after substitution let us say 5 a.

Now, for small rotations that is theta being very very small, sin theta can be said as theta. Then when the tower is in undisplaced position, k will be equal to because now in the above equation theta will get cancelled let us write down that equation for more clarity. So, I should say rho w g summation k equals 1 to n Sk V k minus summation there is theta here summation k equals 1, S k w k theta equal to k of 1 plus t theta. So, for small values of theta I write the equation, now theta gets cancelled and in undisplaced position you will see that this value will also be set to 0. So, now, k will be actually equal to rho w g summation of k equals 1 to n 0 Sk V k minus summation k equals 1 to N S k w k. Now you may introduce a new term which is n 0 let say where n 0 is total number of segments when the tower is undisplaced ok.

(Refer Slide Time: 06:37)

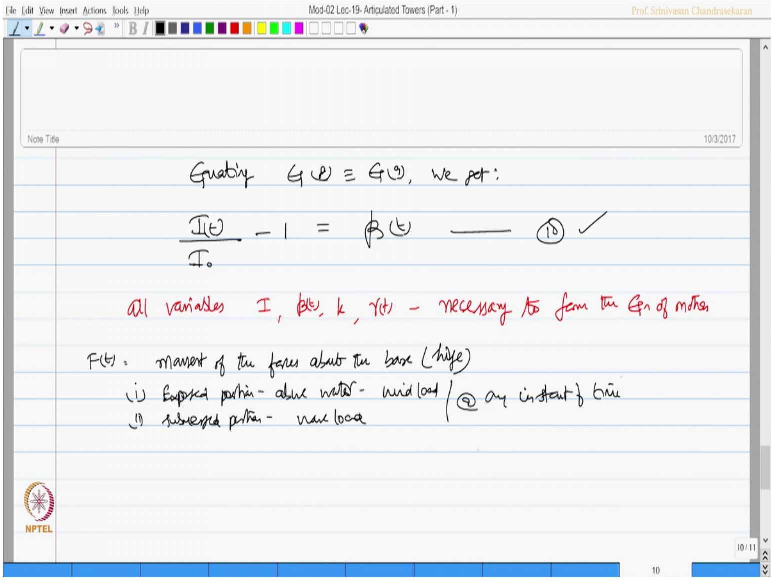


So, at any instant of time gamma t can be computed as R t by k theta minus 1 equation 7. So, gentlemen please look at equation 6 which is this equation.

Its gives me, k equation 7 gives me R of t. Please note that r of t depends on both theta and n at any instant of time, hence it is displacement dependent. Let us talk about mass moment of inertia of the tower about the base. So, which is equal to I t which can be summation of k equals 1 to N, w k by g, S k square because mass moment to inertia about the base. So, second moment of area; plus rho w summation k equals 1 to n 0, C m minus 1 that is variable submergence S k square into V k. So, that g is divided here, so rho w g. So, it is g here, now if you look at the equation of motion that value should be actually equal to the inertia component this is inertia component. So, now, this actually should be equal to the inertia component which is I 0 1 plus B t, I call this as equation number 8.

Now, for undisplaced position of the tower, mass moment of inertia is given by I 0 equals summation k equals 1 to N S k square w k by g plus rho w summation k equals 1 n 0, C m minus 1 S k square V k this equation number 9. So, let us equate equation number 8 and 9 ok.

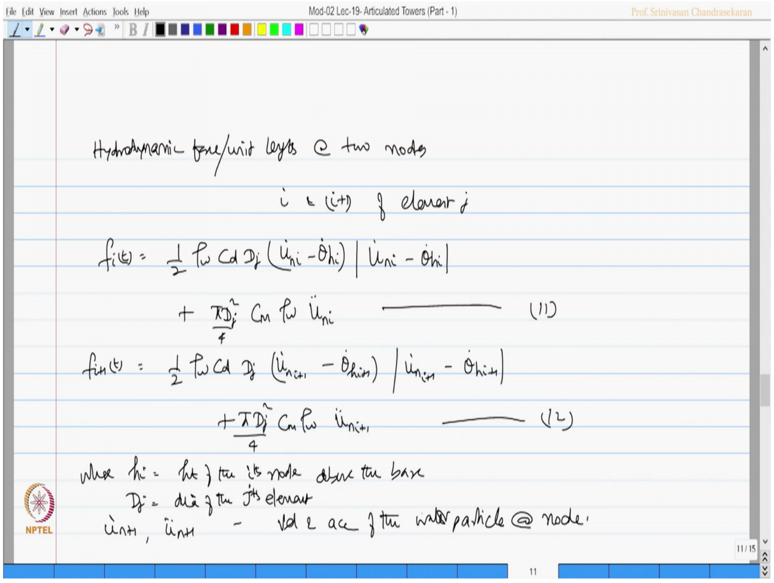
(Refer Slide Time: 09:34)



So, equating equations 8 and 9, we get I t by I naught minus 1 will be B t. So, friends look at this equation we wanted I 0 B t k and gamma t, to qualify the complete equation of motion. So, we got let us say k from equation 6, then I t equation 8, beta t equation 9 and gamma t from earlier equation in equation 7. So, now, we have obtained all variables I, beta t, k gamma t which are necessary to form the equation of motion.

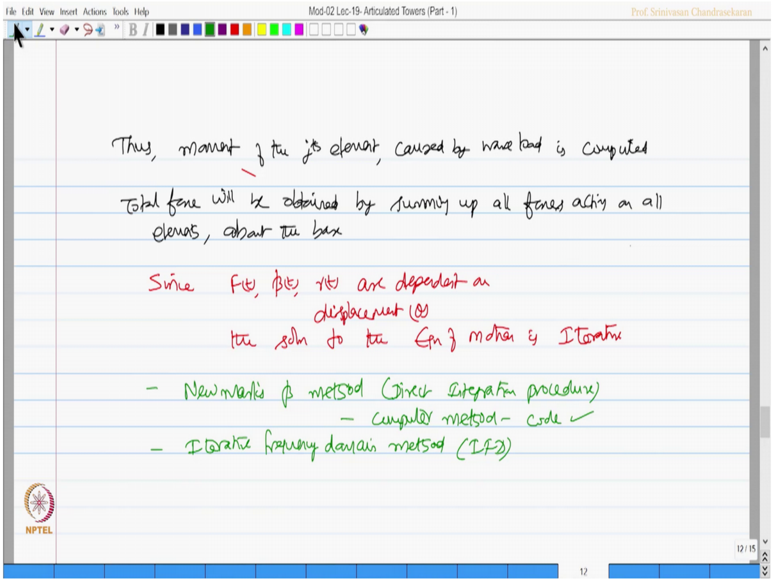
We have got this. So, let us talk about F of t moment of the forces about the base that is about the hinge. So, F of t will have 2 parts, the first part will be exposed portion above water will be subjected to wind load, submerged portion which is underwater will be subjected to wave load at any instant of time.

(Refer Slide Time: 11:38)



Having said this, the hydrodynamic force per unit length at 2 nodes; let us consider 2 nodes node i and i plus 1 of element j. So, f i of t will be half rho w C d D j u dot n i minus theta dot h i absolute, u dot n i minus theta dot h i plus pi d j square by 4 C m rho w u double dot n i let us call this equation number 11. Let us find out the force for i plus oneth node of the j eth element. So, half rho w C d D g u dot n i plus 1 minus theta dot h i plus 1 absolute, u dot n i plus 1 minus theta dot h i plus 1, plus pi D j square by 4 C m rho w u double dot n i plus 1, I call equation number 12 where h I is the height of the i eth node above the base, D j is the dia of the j eth element, u dot n plus 1 u double dot n plus 1 or velocity and acceleration of the water particle at node i.

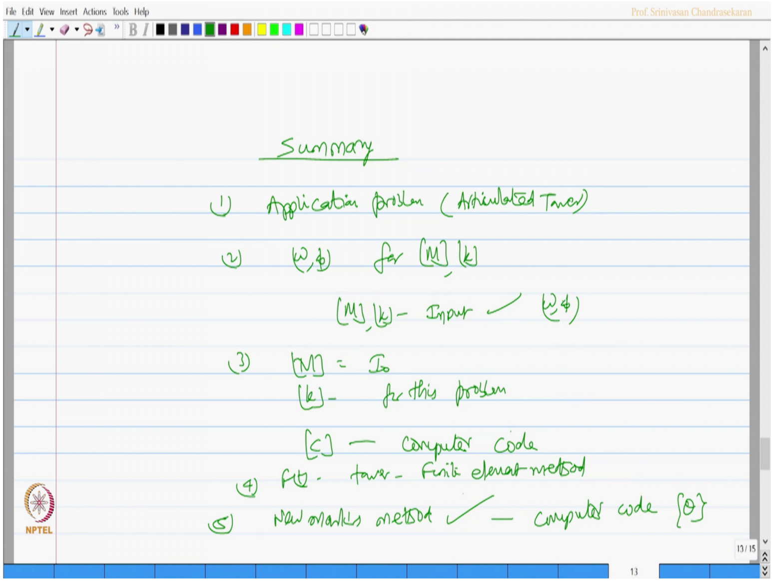
(Refer Slide Time: 13:54)



Thus moment of the j eth element caused by wave load is computed. Total force will be of time by summing up all the forces acting on all elements about the base. Now since F of t, beta of t and gamma of t are dependent on displacement, theta the solution to this equation of motion is iterative. We have already discussed the numerical method for solving iterative solution; we can use either new marks beta method which is one of the direct integration procedure to solve the problem. The computer method for solving this problem the computer code is already available to you.

You can alternately solve this problem using iterative frequency domain method, which is relatively new concept in solving equation of motion of this type ok.

(Refer Slide Time: 15:46)



So, friends in this lecture we learned one application problem on articulated tower, we already have a computer program which calculates omega and phi for the known value of mass and k. So, for mass and k to be input one can find out omega and phi. So, this example explained you how to find out mass, which is as same as I naught and k for this problem. Once you know mass and k, one can compute c using the computer code. The problem also helped you how to estimate the wave forces which is F of t for a given tower using finite element.

So, in the solutions iterative, we already know how to apply new marks method to solve this problem. So, one can easily write a computer code algorithm to solve this problem and try to find the vector theta, with the help of computer codes given in part and parcel.

Thank you very much.