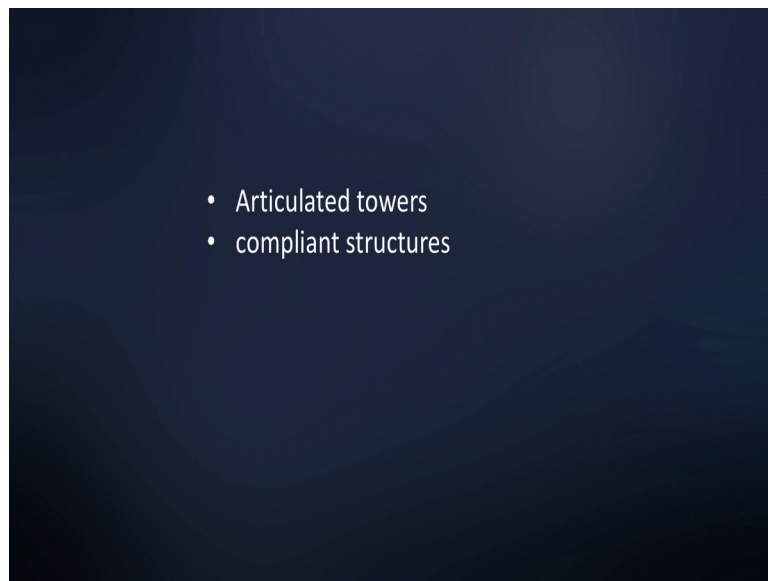


Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 02
Lecture - 19
Articulated Towers (Part - 2)

(Refer Slide Time: 00:16)



(Refer Slide Time: 00:24)

Restoring moment, due to the buoyancy force is given by:

$$M_{\text{restoring}} = \sum_{k=1}^n S_k \sin \theta (V_k \rho_w g) \quad \text{--- (1)}$$

Where n = # submerged elements
 V_k = V of k^{th} element
 ρ_w = density of sea water
 g = g due to gravity (9.81 m/s^2)

$$M_k = M_{\text{restoring}} = \rho_w g \sin \theta \sum_{k=1}^n S_k V_k \quad \text{--- (2)}$$

Restoring moment, (recastering moment) $R(t)$
 $= R(t) = M_k - M_b$

Now, the restoring moment which comes due to the buoyancy force is given by M_{rest} restoring is again $\sum_{k=1}^n S_k \sin \theta$ into $\sum_{k=1}^n V_k \rho_w g$; sum this up for k equals 1 to n where n is the number of submerged elements and V_k is the volume of k th element, ρ_w density of sea water and g is acceleration due to gravity, which is 9.81 meter per second square. So, I call this as equation number 4. In fact, I rewrite equation number 4 straight in a different form as restoring moment is $\rho_w g \sin \theta \sum_{k=1}^n S_k V_k$, I call this as equation 4.

Now I have the (Refer Time: 02:09) turning moment which is $M_{overturning}$, I have the restoring moment which is M_R , now I can always find the resistive moment which is otherwise called as recentering moment. If you recollect it is important to note that **compliant** structures are generally designed to have a very high recentering capability therefore, let us try to find out the recentering moment which is given as R_t , which will be actually equal to the restoring moment minus overturning moment.

(Refer Slide Time: 02:54)

The image shows a digital whiteboard with the following handwritten content:

$$R(t) = M_{rest} - M_{over} = \text{restoring force/moment}$$

$$= k(1 + r(t))\theta \quad (5)$$

Substituting from Eq. (4) & (3) is Eq. (5) we get:

$$\sin\theta \left(\rho_w g \sum_{k=1}^n S_k V_k \right) - \sin\theta \sum_{k=1}^n S_k W_k = k(1 + r(t))\theta \quad (5a)$$

for small rotation ($\theta \ll 1$) $\sin\theta \approx \theta$.
 when the tower is in undisturbed position.

$$\cancel{\theta} \rho_w g \sum_{k=1}^n S_k V_k - \cancel{\theta} \sum_{k=1}^n S_k W_k = k(1 + r(t))\cancel{\theta}$$

$$\checkmark k = \rho_w g \sum_{k=1}^n S_k V_k - \sum_{k=1}^n S_k W_k \quad (6)$$

where n_0 = total # of segments, when $k=1$ for tower is undisturbed.

The restoring moment minus overturning moment if I call this as R_t from a original equation of motion, please look at the original equation of motion from the original equation of motion this is my restoring force.

(Refer Slide Time: 03:08)

Equation of motion will be given by:

$$I_0 (1 + \beta \epsilon) \ddot{\theta} + 2 \sum I_0 \omega_n \dot{\theta} + k (1 + r \epsilon) \theta = F \epsilon - U$$

Rearranging the above Eq. we get:

$$I_0 \ddot{\theta} + 2 \sum I_0 \omega_n \dot{\theta} + k \theta = F \epsilon - k r \epsilon \theta - I_0 \beta \epsilon \ddot{\theta} - U$$

Since the structure is compliant, which undergoes (permitted to undergo) large displacement, RHS force value is reduced.

- This is one of the main advantage of compliant systems
- Form-based design
- displacements are important; not the shapes

So, this should be equal to be restoring force in fact, restoring moment because I am taking moment above the hinge in all the cases. So, I should say this should be actually equal to k of 1 plus r of t into θ . So, now, I substituting, this is going to be equation 5 now substituting from equation 3 and equation 4 in equation 5, we get $\sin \theta \rho \omega g \sum_{k=1}^n S_k V_k$ minus $\sin \theta \sum_{k=1}^N S_k w_k$ is actually equal to k 1 plus r t of θ . So, I call now this as equation 5 after substitution let us say 5 a.

Now, for small rotations that is θ being very very small, $\sin \theta$ can be said as θ . Then when the tower is in undisplaced position, k will be equal to because now in the above equation θ will get cancelled let us write down that equation for more clarity. So, I should say $\rho \omega g \sum_{k=1}^n S_k V_k$ minus $\sum_{k=1}^N S_k w_k \theta$ here θ equal to k of 1 plus t θ . So, for small values of θ I write the equation, now θ gets cancelled and in undisplaced position you will see that this value will also be set to 0. So, now, k will be actually equal to $\rho \omega g \sum_{k=1}^n S_k V_k$ minus $\sum_{k=1}^N S_k w_k$. Now you may introduce a new term which is n_0 let say where n_0 is total number of segments when the tower is undisplaced ok.

(Refer Slide Time: 06:37)

@ any instant of time,

$$\gamma(t) = \frac{R(t)}{k\theta} - 1 \quad \text{--- (7)}$$
 $\gamma(t)$ depends on both θ & n @ any instant of time
 Hence the variable is displacement dependent
 Mass MoI of the tower about the base

$$I(t) = \sum_{k=1}^N \left(\frac{W_k}{g}\right) s_k^2 + \rho_w \sum_{k=1}^{n_0} (C_m - 1) s_k^2 V_k \equiv I_0 (1 + \beta(t)) \quad \text{--- (8)}$$
 for undisplaced position of the tower, Mass MoI is given by

$$I_0 = \sum_{k=1}^N s_k^2 \left(\frac{W_k}{g}\right) + \rho_w \sum_{k=1}^{n_0} (C_m - 1) s_k^2 V_k \quad \text{--- (9)}$$

So, at any instant of time $\gamma(t)$ can be computed as $R(t)$ by $k\theta$ minus 1 equation 7. So, gentlemen please look at equation 6 which is this equation.

Its gives me, k equation 7 gives me R of t . Please note that r of t depends on both θ and n at any instant of time, hence it is displacement dependent. Let us talk about mass moment of inertia of the tower about the base. So, which is equal to $I(t)$ which can be summation of k equals 1 to N , w_k by g , S_k square because mass moment to inertia about the base. So, second moment of area; plus ρ_w summation k equals 1 to n_0 , C_m minus 1 that is variable submergence S_k square into V_k . So, that g is divided here, so $\rho_w g$. So, it is g here, now if you look at the equation of motion that value should be actually equal to the inertia component this is inertia component. So, now, this actually should be equal to the inertia component which is $I_0 (1 + \beta(t))$, I call this as equation number 8.

Now, for undisplaced position of the tower, mass moment of inertia is given by I_0 equals summation k equals 1 to N S_k square w_k by g plus ρ_w summation k equals 1 to n_0 , C_m minus 1 S_k square V_k this equation number 9. So, let us equate equation number 8 and 9 ok.

(Refer Slide Time: 09:34)

The screenshot shows a presentation slide with the following content:

Equating $E \psi \equiv E \psi$, we get:

$$\frac{I(t)}{I_0} - 1 = \beta(t) \quad \text{--- (8) ✓}$$

All variables $I, \beta(t), k, \gamma(t)$ - necessary to form the Eqn of motion

$F(t)$ - moment of the forces about the base (hinge)

- (i) Exposed portion - above water - wind load / @ any instant of time
- (ii) Submerged portion - wave load

The slide also features a toolbar at the top, a title bar with 'Mod-02 Lec-19 Articulated Towers (Part - 1)', and an NPTEL logo at the bottom left.

So, equating equations 8 and 9, we get $I(t) - I_0$ will be $B(t)$. So, friends look at this equation we wanted $I_0, B(t), k$ and $\gamma(t)$, to qualify the complete equation of motion. So, we got let us say k from equation 6, then $I(t)$ equation 8, $\beta(t)$ equation 9 and $\gamma(t)$ from earlier equation in equation 7. So, now, we have obtained all variables $I, \beta(t), k, \gamma(t)$ which are necessary to form the equation of motion.

We have got this. So, let us talk about $F(t)$ moment of the forces about the base that is about the hinge. So, $F(t)$ will have 2 parts, the first part will be exposed portion above water will be subjected to wind load, submerged portion which is underwater will be subjected to wave load at any instant of time.

(Refer Slide Time: 11:38)

Hydrodynamic force/unit length @ two nodes
 i & $(i+1)$ of element j

$$f_i(t) = \frac{1}{2} \rho_w C_d D_j (\dot{u}_{ni} - \dot{\theta} h_i) | \dot{u}_{ni} - \dot{\theta} h_i | + \frac{\pi D_j^2}{4} C_m \rho_w \ddot{u}_{ni} \quad (11)$$

$$f_{i+1}(t) = \frac{1}{2} \rho_w C_d D_j (\dot{u}_{n,i+1} - \dot{\theta} h_{i+1}) | \dot{u}_{n,i+1} - \dot{\theta} h_{i+1} | + \frac{\pi D_j^2}{4} C_m \rho_w \ddot{u}_{n,i+1} \quad (12)$$

where $h_i = h_i$ the i th node above the base
 $D_j =$ dia of the j th element
 $\dot{u}_{ni}, \ddot{u}_{ni}$ - vel & acc of the water particle @ node i .

Having said this, the hydrodynamic force per unit length at 2 nodes; let us consider 2 nodes node i and i plus 1 of element j . So, f_i of t will be half $\rho_w C_d D_j u \dot{n}_i$ minus $\theta \dot{h}_i$ absolute, $u \dot{n}_i$ minus $\theta \dot{h}_i$ plus $\frac{\pi D_j^2}{4} C_m \rho_w u \ddot{n}_i$ let us call this equation number 11. Let us find out the force for i plus one node of the j th element. So, half $\rho_w C_d D_j u \dot{n}_{i+1}$ minus $\theta \dot{h}_{i+1}$ absolute, $u \dot{n}_{i+1}$ minus $\theta \dot{h}_{i+1}$, plus $\frac{\pi D_j^2}{4} C_m \rho_w u \ddot{n}_{i+1}$, I call equation number 12 where h_i is the height of the i th node above the base, D_j is the dia of the j th element, $u \dot{n}_{i+1}$ or velocity and acceleration of the water particle at node i .

(Refer Slide Time: 13:54)

Thus, moment of the j 'th element, caused by wave load is computed

Total force will be obtained by summing up all forces acting on all elements, about the base

Since $F(t)$, $\beta(t)$, $\gamma(t)$ are dependent on displacement (θ) the soln to the Eqn of motion is Iterative

- Newmark's β method (Direct integration procedure)
 - Computer method - code ✓
- Iterative frequency domain method (IFD)

Thus moment of the j th element caused by wave load is computed. Total force will be of time by summing up all the forces acting on all elements about the base. Now since F of t , β of t and γ of t are dependent on displacement, θ the solution to this equation of motion is iterative. We have already discussed the numerical method for solving iterative solution; we can use either new marks beta method which is one of the direct integration procedure to solve the problem. The computer method for solving this problem the computer code is already available to you.

You can alternately solve this problem using iterative frequency domain method, which is relatively new concept in solving equation of motion of this type ok.

(Refer Slide Time: 15:46)

The image shows a digital whiteboard with handwritten notes in green ink. The notes are organized into a list of five points, each starting with a circled number. The text is as follows:

- 1) Application problem (Articulated Tower)
- 2) (ω, ϕ) for (M, k)
 (M, k) - Input ✓ (ω, ϕ)
- 3) $[M] = I_0$
 $[k]$ - for this problem
 $[c]$ - computer code
- 4) FE - tower - finite element method
- 5) Newmarks method ✓ - computer code $\{ \theta \}$

The whiteboard interface includes a menu bar at the top with options like 'File', 'Edit', 'View', 'Insert', 'Actions', 'Tools', and 'Help'. A toolbar with various drawing tools is visible below the menu. The name 'Prof. Srinivasan Chandrasekaran' is written in the top right corner. An NPTEL logo is in the bottom left corner, and the number '13' is in the bottom right corner.

So, friends in this lecture we learned one application problem on articulated tower, we already have a computer program which calculates omega and phi for the known value of mass and k. So, for mass and k to be input one can find out omega and phi. So, this example explained you how to find out mass, which is as same as I naught and k for this problem. Once you know mass and k, one can compute c using the computer code. The problem also helped you how to estimate the wave forces which is F of t for a given tower using finite element.

So, in the solutions iterative, we already know how to apply new marks method to solve this problem. So, one can easily write a computer code algorithm to solve this problem and try to find the vector theta, with the help of computer codes given in part and parcel.

Thank you very much.