

Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 02
Lecture - 20
Tension Leg platforms

(Refer Slide Time: 00:17)

- Tension leg platform
- Problem formulation
- Derivation of stiffness matrix from first principle

(Refer Slide Time: 00:23)

File Edit View Insert Actions Tools Help Mod-02 Lec-20- Tension for Platform (TLP) (Part - 2) Prof. Srinivasan Chandrasekaran

Increase in tension, $\Delta T_i = \left[\sqrt{x_i^2 + l^2} - l \right] \frac{AE}{l}$

where A = cross-sectional area of tether
 E = Modulus of elasticity of tether material
 x_i = Arbitrary displacement in surge degree-of-freedom (with to derive kit)
 l = length of the tether (unstretched)
 ΔT_i = increase in tension, due to arbitrary displacement (x_i)

Eqn of force in surge direction stiffness is N/m $\frac{F_{sur}}{\Delta}$

$$K_{11}(x_i) = 4(T_0 + \Delta T_i) \sin^2 \alpha \quad (1)$$

$$\sin^2 \alpha = \frac{x_i^2}{x_i^2 + l^2} \quad (2)$$

sub(2) in (1), we get

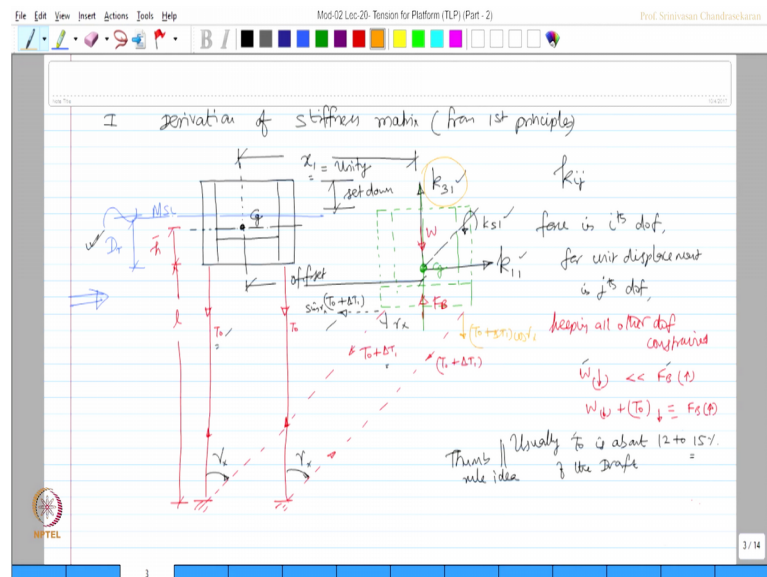
$$K_{11} = \frac{4(T_0 + \Delta T_i)}{\sqrt{x_i^2 + l^2}} \quad (3)$$

4/14

Delta T 1 as x^2 square plus l^2 square the whole root minus l multiplied by AE by l , where A is the cross sectional area of tethers E is modulus of elasticity of tether material if it is T we already know the value of steel x is the arbitrary displacement in surge degree of freedom, l is the length of the tethers, tethers are considered to be **inextensible**.

So, delta T 1 is increase is tension due to arbitrary displacement x to be very clear this arbitrary displacement is considered as unity to derive the stiffness matrix. Now having said this let us write down the equilibrium of forces in surge direction, I request you to refer to this figure again.

(Refer Slide Time: 02:33)



There are forces acting in the surge direction k_{11} and there is an component of this in the x axis, if this is γx this value is also γx .

Obviously, I should say this will be T_0 plus ΔT of $\sin \gamma$, x because the vertical component will be $\cos \gamma$ x and this is $\sin \gamma$ x ; so k_{11} and this. So, our opposites in direction as you see from this figure. So, write the equilibrium of these forces you know stiffness units actually are in newton per meter that is force per by displacement.

So, if you want to find the force I should multiply the stiffness coefficient by the displacement to get the force, I am writing the force equilibrium therefore, I should say k_{11} into x it should be equal to we can see here this force will compromise, which will

be $T_0 + \Delta T_1 \sin \gamma_x$, but friends please see TLP has got 4 legs or 4 columns each column will have a set of tethers.

So, each tether value is T_0 there will be 4 such legs, further from the figure $\sin \gamma_x$ is actually equal to x_1 by root of $x_1^2 + l^2$ you can see from this geometry $\sin \gamma_x$ will be x_1 by root of this value this value. So, now, substituting this let us say call this equation 1 and equation 2 substituting 2 in 1, we get k_{21} as 4 times of $T_0 + \Delta T_1$ by root of $x_1^2 + l^2$ equation 3.

So, this is my coefficient which obtained from the surge displacement, naturally no force in sway direction.

(Refer Slide Time: 05:00)

no force in sway direction. $k_{21} = 0$. (4)

Eqn of forces in heave direction

$$(k_{31})(\Delta) = 4(T_0 \cos \gamma_x + \Delta T_1 \cos \gamma_x) - 4T_0 \quad (5)$$

$$\cos \gamma_x = \frac{l}{\sqrt{x_1^2 + l^2}} \quad (6)$$

sub (6) in (5) we get

$$k_{31}(\Delta) = 4T_0 (\cos \gamma_x - 1) + 4\Delta T_1 \cos \gamma_x$$

$$k_{31} = \frac{4T_0 (\cos \gamma_x - 1) + 4\Delta T_1 \cos \gamma_x}{\Delta} \quad (7)$$

where $\Delta = \text{set-down}$

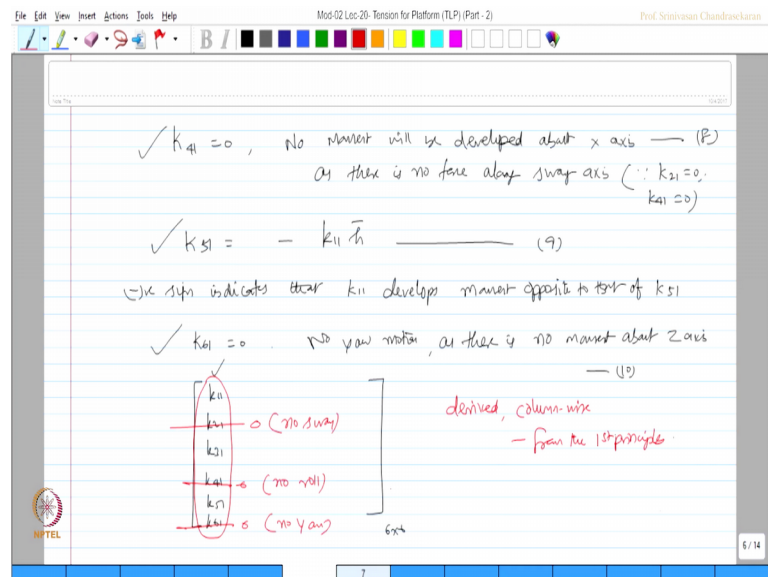
And therefore, k_{21} will be 0, let us talk about equilibrium of forces in heave direction. So, I am talking about equilibrium in the vertical axis now heave direction. So, you see k_{31} will be 1 component k_{31} will be 1 component in the heave direction and the vertical component of this will be $T_0 + \Delta T_1 \cos \gamma_x$.

So, now I want to write the equilibrium for this which will be $k_{31} \Delta$ will be equal to $T_0 \cos \gamma_x + \Delta T_1 \cos \gamma_x$ there are 4 such legs minus initial tension of 4 because I am looking only for the difference right. Also from the geometry $\cos \gamma_x$ is actually l by root of $x_1^2 + l^2$. So, substituting back this is equation number let us say 4.

This equation number 5 this become 6. So, substituting equation 6 in equation 5 we get k_{31} into delta will be $4T_0 \cos \gamma x \sin \delta$ plus $4\delta T_1 \cos \gamma x$ again substituting for $\cos \gamma x$, we say $4T_0 \cos \gamma x \sin \delta$ plus $4\delta T_1 \cos \gamma x$ by delta this becomes k_{31} which is going to be a 7.

Now, you know the value of $\cos \gamma x$ and substitute them and write in terms of l by x^2 plus l square now what is this delta? This delta is nothing, but the set down. So, delta is the set down effect. So, naturally when we apply a force along x axis the rotation about x axis will be 0.

(Refer Slide Time: 07:58)



So, k_{41} will be 0 because no moment will be developed about x axis as there is no force along sway axis that is since k_{21} is 0 k_{41} will be 0 ok.

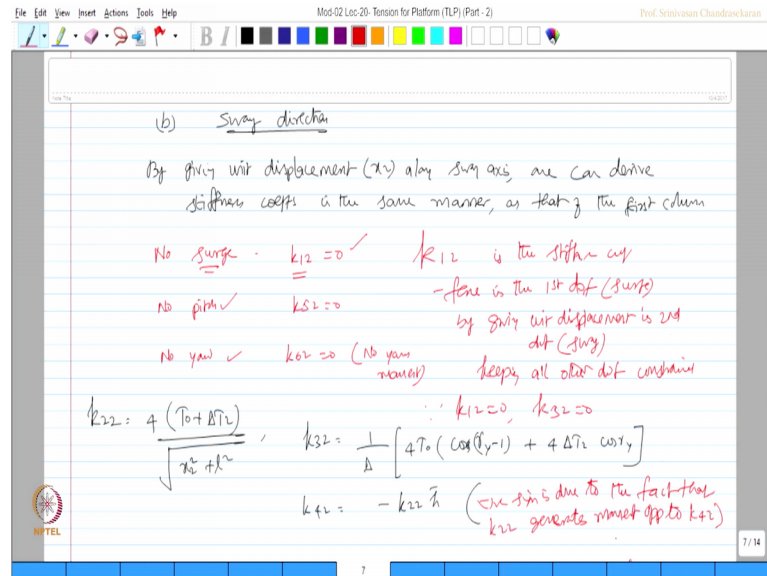
Let us say like this. So, if I call this as equation number 8 let us talk about k_{51} the rotation this is going to be this is anti-clockwise about C_g let us take the moment about the keel right. So, this distance is \bar{h} . So, I should say k_{51} should be actually equal to minus of k_{11} into \bar{h} . So, this is equation number 9 the negative sign indicates that k_{11} develops moment opposite to that of k_{51} and k_{61} will be 0.

No yaw motion because as there is no moment about z axis. So, that is equation number 10. So, now, if you look at friends we have k_{11} we have k_{21} , we have k_{31} , we have k_{41} we have k_{51} , k_{61} . So, now, in the k matrix of 6 by 6 the first column k_{11} one k_{21}

1 k 3 1, k 4 1, k 5 1, k 6 1 are now ready please understand stiffness matrix is always derived column wise.

So, the first column of the stiffness matrix is now derived from the first principles similarly we can do for the sway direction.

(Refer Slide Time: 11:09)



So, look at this figure if you look at this figure we have given displacement x_1 along surge. So, now, I should give displacement x_2 along sway where sway is along the y axis. So, having said this by giving unit displacement x_2 along sway axis, one can derive stiffness coefficients in the same manner as that of the first column.

But let us keep a very important I on the non-zero elements or let us say 0 elements if you look at the first column we already said that k_{21} is 0, k_{41} is 0, and k_{61} is 0, k_{21} is 0, k_{41} is 0 and k_{61} is 0 it means no sway no roll and no yaw on the same basis I should say no surge, no pitch and no yaw no surge that is k_{12} will be 0 please understand how I explained k_{12} k_{12} is the stiffness coefficient that is force in the first degree of freedom, that is surge by giving unit displacement in second degree of freedom that is sway keeping all other degrees of freedom constrained.

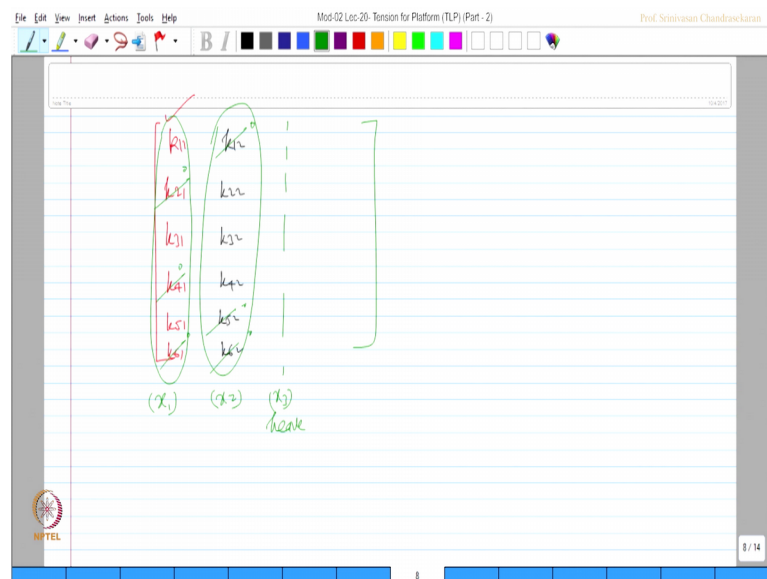
So, k_{12} since there is no surge this will be 0. So, I should say $k_{12} = 0$ since k_{12} is 0 I can expect that k_{52} will also be 0 it means no pitch. So, k_{52} will also be 0 and of course, k_{62} will be 0 because no yaw moment. Now I am interested in finding out k_{22}

which is similar to that of k_{11} see a $k_{11} T_0 \Delta T_1$ by this similarly I can write k_{22} as T_0 plus ΔT_2 divided by square root of x_2^2 plus l^2 4 times you can see here.

I have only replaced this and this the second suffixes 2 done this. Once I know k_{22} I can now find k_{32} also. So, that is going to be 1 by $\Delta T_0 \cos \gamma$ minus 1 plus $4 \Delta T_2$ of $\cos \gamma$ and k_{42} will be minus k_{22} of h bar now one can write negative sign is due to moment due to the fact that k_{22} generates moment opposite to k_{42} ok.

Now, I have got the second column you know k_{12} , k_{22} , k_{32} , k_{42} , k_{52} and k_{62} I got the second column.

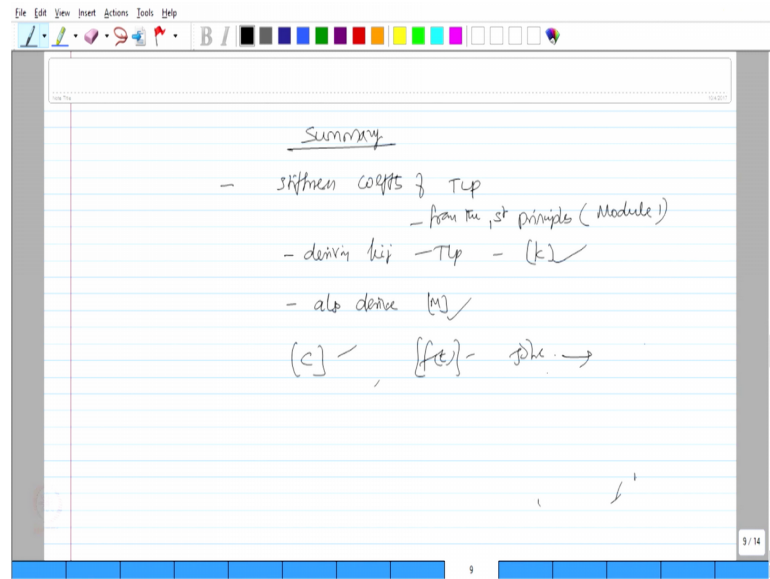
(Refer Slide Time: 16:03)



So, I can now say I have got the second column k_{12} , k_{22} , k_{32} , k_{42} , k_{52} and k_{62} I already have now I have k_{11} , k_{22} , k_{33} , k_{44} , k_{55} and k_{66} out of which let us mark the zeros this is 0 this was 0 and this was 0.

Similarly, this is 0, this was 0, and this is 0. So, first column is completed second column is completed by giving unit displacement along surge you know displacement along sway let us go for the third column. So, what should I do? I should give unit displacement along heave that is good. So, let us do this we will continue with the discussion in the next lecture.

(Refer Slide Time: 17:06)



So, the summary friends we are attempting to derive the stiffness coefficients of a tension leg platform from the first principles, which we learn from module 1 we are deriving the stiffness coefficients of a TLP to get the stiffness matrix. We will also derive the mass matrix. Once I have the mass and stiffness matrix I can find the damping matrix I can find f of t . And then I can solve this problem which I will explain in the subsequent lectures.