

Computer Methods of Analysis of Offshore Structures
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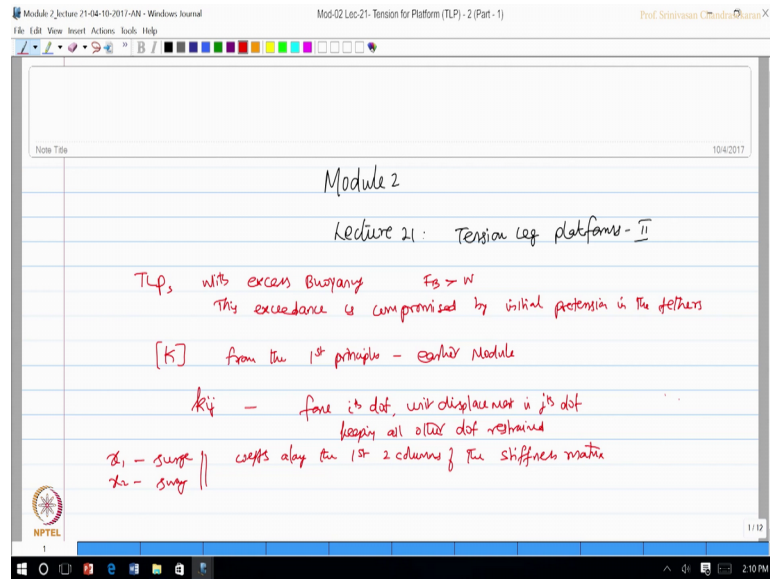
Module – 02
Lecture – 21
Tension Leg platforms - 2

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- Tension leg platform
- Rotational degree of freedom
- Stiffness matrix of TLP

So, friends let us continue the discussion on derivation of stiffness and mass matrix from the first principles for an example problem of an offshore structure. We have considered tension leg platforms in the last lecture we discussed about how conceptually TLPs are actually designed and developed.

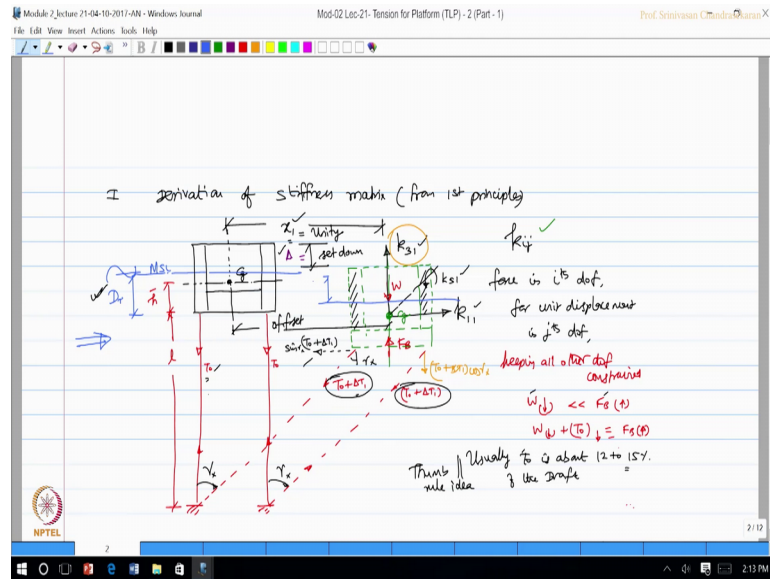
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To recollect that let us see make this statement TLPs or platforms with excess buoyancy, that is buoyancy exceeds the weight of the platform and this is compromised by initial pretension in the tethers.

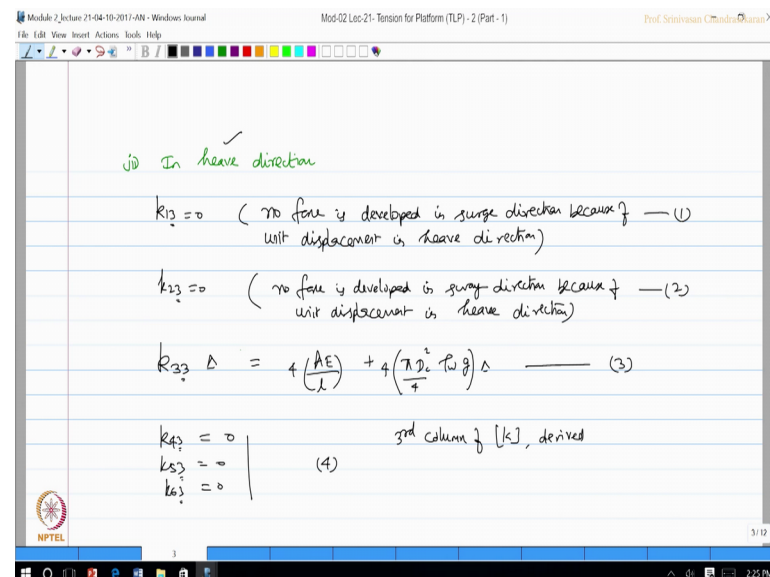
We wanted to develop or derive the stiffness matrix for TLP from the first principles, what we learn from the earlier lectures and earlier module, we are developing the stiffness coefficients shaking that we are trying to find out the force in i th degree by giving unit displacement, in j th degree keeping all other degrees of freedom restrained. So, we give unit displacement in surge, we give unit displacement in sway and we developed coefficients along the first 2 columns of the stiffness matrix.

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Now, let us derive the stiffness matrix for the heave motion. So, the same figure we have the platform given with an offset and set down which is happening in this figure, we already said what is our k_{ij} . So, we should give now unit displacement in the heave direction and try to find forces in all effective degrees of freedom keeping other degrees of freedom restrained.

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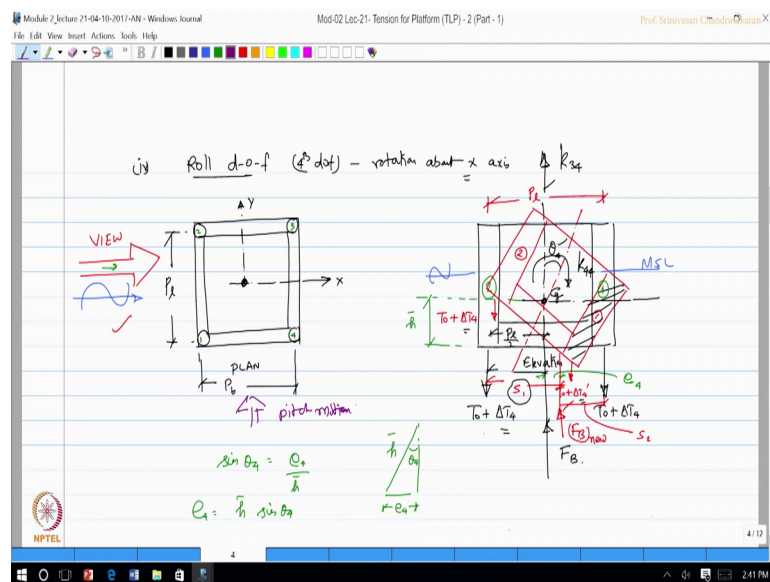
So, let us say in heave direction k_{13} will be 0 because no force is developed in surge direction because of unit displacement in heave direction. Let us say this is again

equation number 1 similarly k_{23} is 0 for the same reason no force is developed in sway direction because of unit displacement in heave direction call equation 2, now I want to find what will be the force in the heave direction because of unit displacement given in the heave direction. So, k_{33} delta if you look at the figure now when you give heave displacement there is a set down happening and that will now cause the change in tether tension. So, k_{33} delta will be now AE by 1 of 4 tethers plus these column members will undergo yeah variable submergence effect for example, let us plot this m s l here.

So, this is γ_1 by which an extra submergence happens. So, I should say ρ diameter of the column a^4 into ρ w into g into δ . So, that gives me the volume and yes gives in the force now, and there are force actually x . So, we call this equation number 3. Now k_{43} , k_{53} and k_{63} will be 0. So, now, I have the third column of the stiffness matrix because the second subscript indicates unit displacement given, along the third degree of freedom which is heave and I get forces in almost all degrees of freedom as 1 2 3 4 5 and 6.

So, I have now the third column of the stiffness matrix derived.

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Now, let us go to roll degree of freedom. So, that is the fourth degree of freedom this is rotation about x axis. Let us say I have a TLP whose deck consists of 4 column members connected by the pontoon members on the top, as well as the bottom. So, this is the plan I am drawing. So, let us say the C g of this, this is my x axis this is my y axis in plan in

plan and is dimension I call this as plan breath and this as plan length and this is my wave direction.

Now, I want to name these column members as 1 2 3 and 4, we all know that roll is rotation about x axis. So, I what to rotate it about x axis let us view this figure this object from this direction and draw it here. So, when I try to draw it here, original elevation will be the corner members in the top deck and the bottom pontoon. This will be elevation now this is elevation.

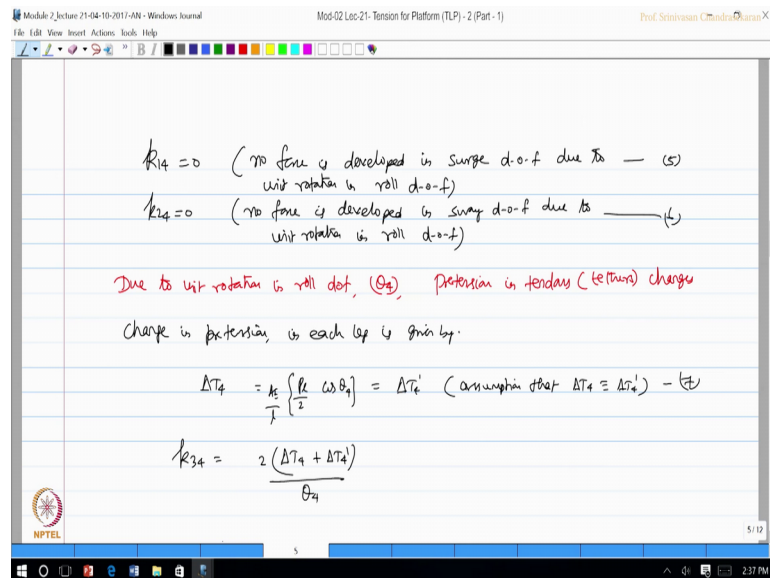
So, naturally this member will be 2 and this member will be 1 because I am viewing this from this direction. Now the C g will be located somewhere here, this is my C g and this is my water level which is says mean sea level. Now I want to roll this about x axis so; obviously, the new position of the body will be like this let us roll it by this way.

So, this becomes my column 2 this becomes my column 1 column 1, initially I had some tension in these legs which I call as T_0 plus delta T_4 , 4 stands for the fourth degree of freedom this is T_0 it as delta T_4 . Now, there will be new tension in these legs these values will be T_0 plus delta T_4 and T_0 plus delta T_4 dash.

So, the original buoyancies enter was exactly acting here, now the buoyancy will be shifted because this column will be more submerged than column 2 therefore, there is shift in buoyancy this is a new buoyancy force centre from that line of action let this force or this distance $b_s 1$ and let e be the eccentricity between these 2 that is between the old and the new let this be e and let this distance this distance $b_s 2$.

Now, looking at this figure one can guess the centered center dimension between one and 2 is $pl s pl$. Now we are given unit rotation of θ_4 which is roll the forces could be which is acting as k_{34} and the rotation which is k_{44} let us try to derive this.

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So, k_{14} will be 0 because no force is developed in surge degree, due to unit rotation in roll degree. Let us call equation number k_{24} will also be 0 because no force is developed in sway degree due to unit rotation in roll degree.

Due to unit rotation in roll degree that is θ_4 pretension in the tendons are otherwise called as tethers changes. So, change in pre tension in each leg is given by if you look at this figure, I am talking about change in pretension in each leg is given by ΔT_4 which is actually equal to you can see here this dimension will be pl by 2 because this is pl . So, I should say pl by 2 $\cos \theta_4$ because this angle is θ_4 right. So, $\cos \theta_4$ multiplied by AE by l , we assume that the change in pretension between the near leg and the further leg.

That is ΔT_4 and $\Delta T_4'$ both are same is an assumption. Now look at this figure this value which is the shift between the new and the old C_g I call this as e_4 . So, from this figure I can easily say if this is h bar then I can if this is my h bar and this gives me θ_4 then I can say this distance is actually e_4 . So, with this logic we can now say that k_{34} will be equal to that is equation number 7.

k_{34} will be now equal to $\Delta T_4 + \Delta T_4'$. So, twice of that total is 4 numbers divided by θ_4 . So, from the figure we can easily say $\sin \theta_4$ is e_4 by h bar and hence e_4 is h bar $\sin \theta_4$.

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$$S_1 = \frac{l}{2} + e_4$$

$$S_2 = \frac{l}{2} - e_4 \quad \therefore S_1 + S_2 = l$$

$$F_b = 4 \left\{ \frac{\pi d_c^2}{4} \rho_w g (h) \right\}$$

$$k_{44} = 4 \left\{ \frac{\pi d_c^2}{4} \rho_w g l \sin \theta_0 \right\} + 4 \left\{ T_0 h \sin \theta_0 \right\} + 4 \left\{ \frac{AE}{l} \left(\frac{l}{2} \right) \cos \theta_0 \right\} \quad \text{--- (9)}$$

$$\left. \begin{matrix} k_{54} \\ k_{64} \end{matrix} \right\} = 0 \quad \text{because no moments in pitch/yaw developed due to unit rotation in roll dir}$$

Having said this s_1 is actually equal to $p/2 + e_4$ please see this figure; s_1 is actually equal to this distance plus this. So, I can say it is $p/2 + e_4$ and s_2 is $p/2 - e_4$ on the other hand in simple terms $s_1 + s_2$ will be actually p that is what $s_1 + s_2$ will be actually equal to p .

So, now, I want to compute the extra buoyancy force created by this which will be because of the immersed legs where d_c is the diameter of the column member $\rho_w g$ and Δz is the set down and there are 4 such legs. Now I want write k_{44} which will be πd_c^2 by 4 into $\rho_w g$ into $p \sin \theta$ 44 times of this plus $T_0 h \sin \theta$ 44 times of this AE by $p/2 \cos \theta$ 44 times of this.

So, let us say this my equation 9. So, now, I have k_{14} , k_{24} , k_{34} , k_{44} , k_{54} and k_{64} will be 0 because no moments in pitch/yaw developed due to unit rotation in roll degree of freedom this is equation number 10. So, I have got the entire 4th column now on the stiffness matrix.

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Module 2 Lecture 21-04-16-2017-AM - Windows Journal Prof. Srinivasan Umendra

(v) pitch d.o.f (s^b dof)

$k_{15} = 0$ (. . . .) $k_{45} = 0$ (. . . .)

$k_{25} = 0$ (. . . .) $k_{65} = 0$ (. . . .)

$k_{35} = 2 \frac{(\Delta T_5 + \Delta T_5')}{\theta_s}$ s^b column of the $[k]$ is derived

$k_{55} = 4 \left[\frac{\pi D_c^2 \rho_w g P_b \sin \theta_a}{4} \right]$
 $+ 4 \left[T_0 \bar{h} \sin \theta_s \right]$
 $+ 4 \left[\frac{A_c}{2} \frac{P_b}{2} \cos \theta_s \right]$

Similarly, I can do it for pitch degree of freedom which is the fifth degree of freedom. I can straight away write by looking at the same equation same figure to get pitch I should view this from this direction for pitch motion. So, I should rotate this about y axis that is pitch. So, now, I can say k_{15} is 0, I think you know the reason k_{25} is 0. I think you know the reason k_{35} is $\Delta T_5 + \Delta T_5'$ by θ_s twice of this and k_{55} is $5 D c^2$ by $4 \rho_w g p_b \sin \theta_a$ you know this dimension when you look at this will be p_b , $p_b \sin \theta_a$ into 4 plus $4 T_0 \bar{h} \sin \theta_s$ same as this, plus A_c by 1, p_b by 2 $\cos \theta_s$ into 4 times same as this. So, I get k_{55} . Now k_{45} and k_{65} will be 0. Now, I think you will be now the reason for this, now I have the fifth column generated of the stiffness matrix is now derived.