

Computer Methods of Analysis of Offshore Structures
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Module – 02
Lecture – 21
Tension Leg platforms - 2

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- Tension leg platform
- Rotational degree of freedom
- Stiffness matrix of TLP

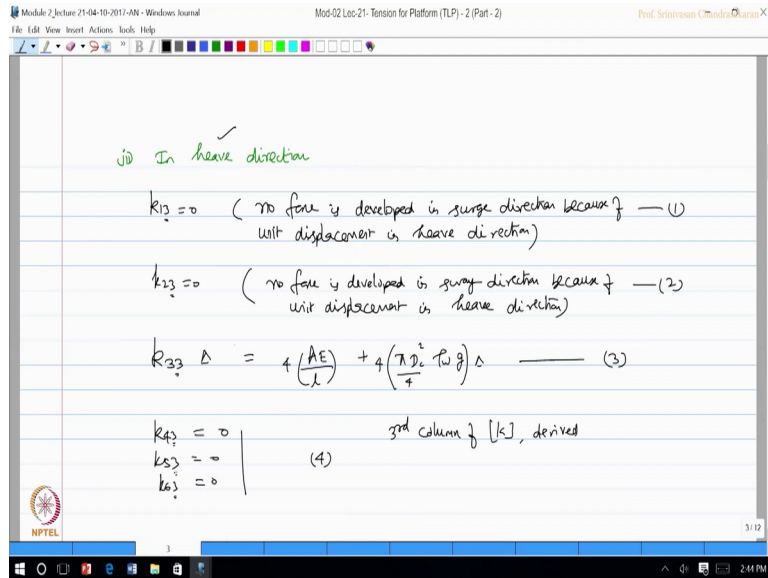
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ω yaw dof (6th dof) — rotation about z axis (yaw)

$$l_1 = \sqrt{l^2 + \theta_2^2 (a^2 + b^2)} \quad (\text{with ref to } f_y, \bar{y})$$

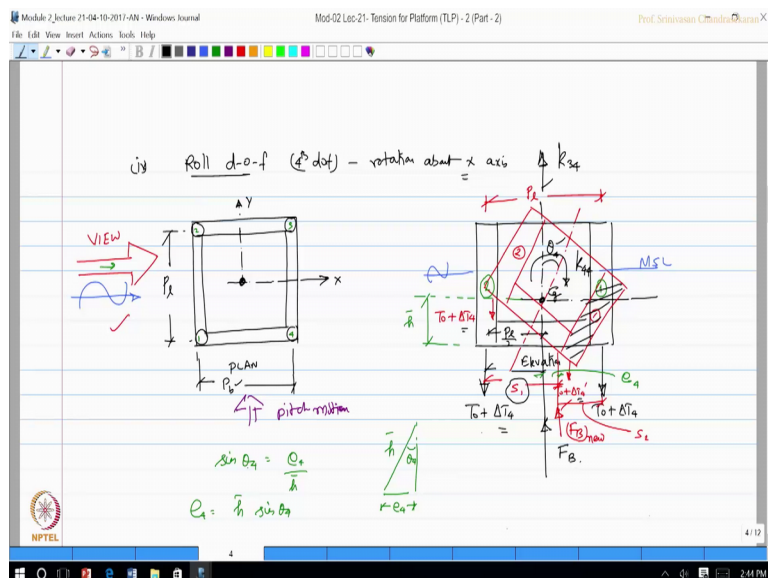
Let us now do for yaw degree of freedom, which is the 6th degree of freedom go back to the plan again this was the original problem this is the plan, I redraw the plan again. So, there are column corners what we have here the columns.

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These columns are numbered in the same manner as we have here 1 2 3 4 same style.

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Let us say 1 2 3 and 4. Let us say I have I am also going to mark the centroidal axis of this, now I am going to give rotation about the z axis that is yaw. So, let me draw the new position let me give the rotation, let me draw the new position. We call this angle as theta

6, let us this is going to be 1 2 3 and 4 I have rotated clockwise, let me call this dimension from here as a and from here this dimension as b, in simple terms d of set with a of set will give me my rotation.

These legs will undergo new tension which will be T_0 plus ΔT_6 , this leg will also now undergo a new tension which will be T_0 plus ΔT_6 and similarly all of them that is how the rotation happens. So, now, the cable length will increase by a small amount which I call as l_1 which will be $l^2 + \theta_6^2 a^2 + b^2$ the whole root with reference to this figure, I call this as equation number.

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Change in pretension in each leg is given by:

$$\Delta T_6 = \frac{AE}{l} [l_1 - l] \quad (1)$$

force in heave direction.

$$k_{36} \theta_6 = 4 T_0 \left(\frac{l}{l_1} - 1 \right) + 4 \Delta T_6 \left(\frac{l}{l_1} \right) \quad (2)$$

$$k_{36} = \frac{4 T_0}{\theta_6} \left[\frac{l}{\sqrt{l^2 + \theta_6^2 (a^2 + b^2)}} - 1 \right] + \frac{4 \Delta T_6}{\theta_6} \left[\frac{l}{\sqrt{l^2 + \theta_6^2 (a^2 + b^2)}} \right] \quad (3)$$

So, change in pretension in each leg is given by ΔT_6 , because rotation is about 6 degree which will be $l_1 - l$ of AE by l that is in each leg this is equation number force in heave direction. Due to this rotation will be k_{36} into θ_6 which will be l by $l_1 - l$ of T_0 of 4 legs plus l by l_1 of ΔT_6 of 4 times, which gives me k_{36} as $4 T_0$ by θ_6 of l by root of $l^2 + \theta_6^2 a^2 + b^2$ minus 1 plus $4 \Delta T_6$ by θ_6 of l pi square root of $l^2 + \theta_6^2 a^2 + b^2$.

I call this is equation number 12 and 13.

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$$K_{66} = 4 \left[\frac{(T_0 + \Delta T_6)(a^2 + b^2)}{L^2 + \theta_6^2 (a^2 + b^2)} \right] \quad (14)$$

Coef. of $[k]$ depends on $(T_0, \Delta T_6)$
 $[k]$ is response dependent
 T_0 - changes as 'when there is a displacement'

K_{66} will be now equal to T_0 plus ΔT_6 of a^2 plus b^2 by $1 + \theta_6^2$ of 4 times which in simple terms 4 times of T_0 plus ΔT_6 a^2 plus b^2 by $1 + \theta_6^2$ of a^2 plus b^2 . I call this is equation number. So, friends let us now assemble the stiffness matrix.

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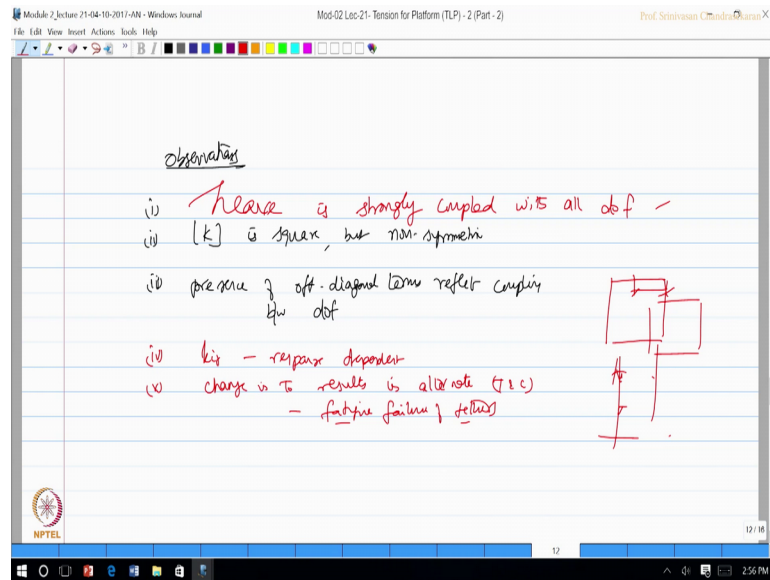
$$[k] = \begin{bmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{22} & 0 & 0 & 0 & 0 \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ 0 & k_{42} & 0 & k_{44} & 0 & 0 \\ k_{51} & 0 & 0 & 0 & k_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{66} \end{bmatrix}$$

6x6 for Truss

Let us say the stiffness matrix derived from first principles is like this, which is k_{11} , k_{22} , k_{31} , k_{32} , k_{42} and 0 .

All values of 0 except k 33 in heave similarly in roll we have k 34 and k 44 remaining all are 0, in pitch we have k 35 and k 55 remaining all are 0, in yaw we have k 36 and k 66 remaining all are 0.

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So, that is my 6 by 6 stiffness matrix for TLP one can very easily see some important observations on this for our understand what are the observations we can see from the stiffness matrix first and foremost you see heave coefficients of present in all it means you give unit displacement in surge sway heave roll pitch and yaw.

You get forces in heave direction in all the case; it means heave is strongly coupled with all degrees of freedom. You take a box which is floating give a heave displacement it will influence surge influence sway everything. So, heave has got of strong coupling the second observation what we have is stiffness matrix is square and non-symmetric. K 36 is present by k 63 is not present for example, k 35 is present k 53 is not present.

So, stiffness matrix is square, but non symmetric. So, it also says that presence of off diagonal terms reflect coupling between degrees of freedom take for example, anyone specific coefficient let us say k 66. So, the coefficients of stiffness matrix depends on tether attention variation and change in tether attention variation you see here. So, I should say that k matrix is response dependent, because T 0 changes only when there is a displacement.

Number 1 change in T_0 k_{ij} is response dependent, change in T_0 results in alternate tension and compression which results in fatigue failure of tethers. So, friends interestingly in this lecture we have learnt how to derive the stiffness matrix from first principles.

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Summary

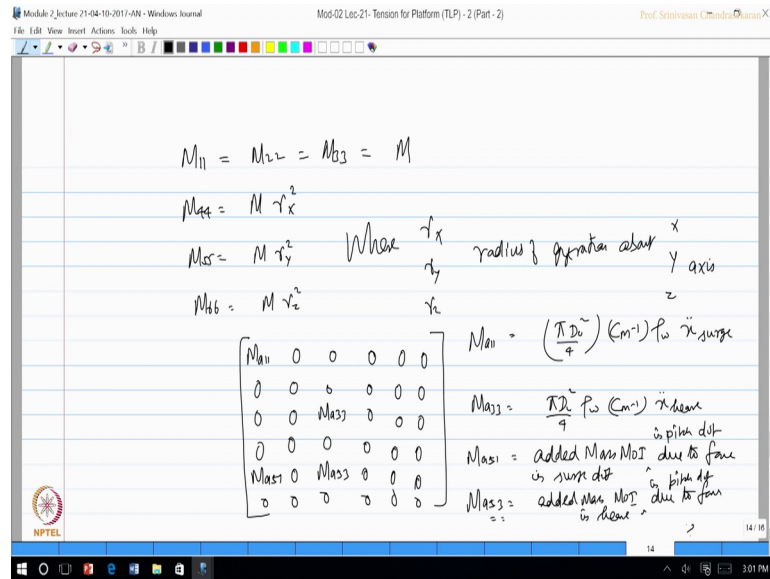
- $[K]$ of TLP is not constant at all times
- coeff change with time step, $-(T_0, x_i)$

$$[M] = \begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix} + \left[\text{added mass} \right]$$

So, one can give a very clear summary the stiffness matrix of TLP is not constant at all times, coefficients change with time step which depends on T_0 and x_i .

Let us see what would be the mass matrix, if this is my mass matrix which is again going to be a square matrix I should say M_{11} 0 0 0 this is also going to be 0 and 0, similarly M_{22} 0 all zeros similarly all zeros with M_{33} then all zeros with M_{44} or M_{22} , M_{33} , M_{44} and all zeros with M_{55} , and all zeros with M_{66} . This is the ideal mass matrix, but some added mass will be added to this.

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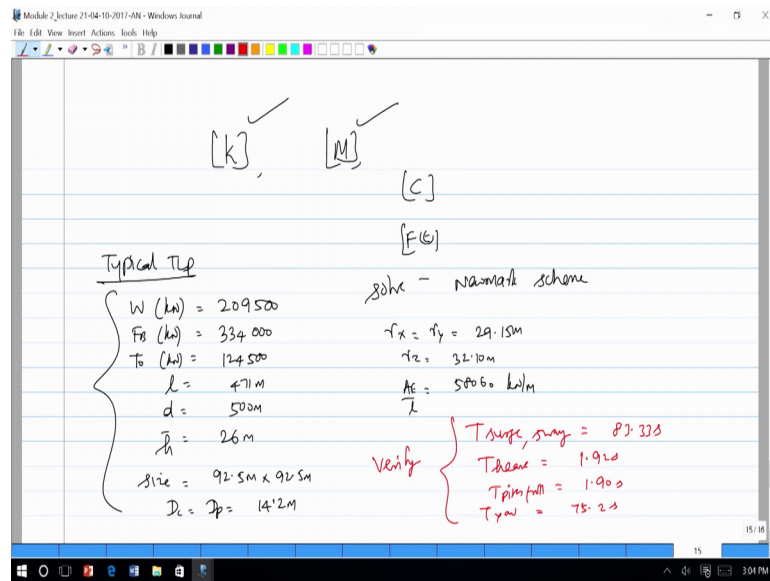
Let us quickly see what is my M_{11} , M_{22} and M_{33} , this will be equal to the mass of the platform. M_{44} will be the mass of the platform into r_x square, M_{55} will be mass of the platform into r_y square and M_{66} will be mass of the platform into r_z square, where r_x , r_y and r_z are respective radius of gyration about x , y and z axis respectively. Now what would be the added mass term? The added mass matrix will be interestingly M_{41} , M_{42} , M_{43} , M_{51} , M_{52} , M_{53} , M_{61} , M_{62} , M_{63} .

Similarly, nothing added in the second column, but in the third column there will be M_{33} , in the fourth column nothing added there will be M_{43} and M_{53} fourth column then fifth column and 6th column. So, what is going to be M_{41} ; M_{41} is going to be $\frac{\pi D_c^2}{4} C M^{-1} \rho_w$ of x dot surge.

Then what is going to be M_{33} which is going to be $\frac{\pi D_c^2}{4} \rho_w$ into r_z square then what is M_{41} and M_{53} . M_{41} is added mass moment of inertia due to force in surge degree similarly M_{53} as you correctly guessed is added mass moment of inertia due to force in heave degree.

So, this is the added mass moment of inertia in pitch degree due to force in surge degree similarly the added mass moment of inertia in pitch degree due to force in heave degree that is why it is M_{53} .

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Now, we have k matrix derived I want to use the computer program to find c matrix and then f of T and then solve the problem by Newmark's iteration scheme which we already gave you in the last lecture. It is a typical TLP very interesting data will have weight in kilo Newton's as 2 0 9 5 0 0 buoyancy force in kilo newton has 3334 triple 0 T 0 in kilo newton it will be 124 500 for a length of 471 meters for a water depth of 500 meters.

The h bar is generally seen as about 26 meters for this platform, the size of the platform is 92.5 meter by 92.5 meters. Diameter of the column and diameter of the pontoon members are about 14.2 meters, r x and r y are 29.15 meter and r z is 32.10 meter and the axial stiffness in cables is about 5 8 0 6 0 kilo newton per meter. There is a typical example TLP, based on which I want you to find out the mass matrix the k matrix run the software or run the coding which I gave you in the previous lectures, you will find that the period of surge and sway will be about 83.33 seconds and period of heave will be about 1.92 seconds and period of pitch and roll will be about 1.9 seconds and period of yaw it will be about 75.2 seconds.

So, please verify these results using the program of I can solve which we gave you in the last lecture. So, this lecture summarizes how to estimate the stiffness matrix, the mass matrix and of course, the damping matrix for a typical TLP as you see on the screen now.

Thank you very much.