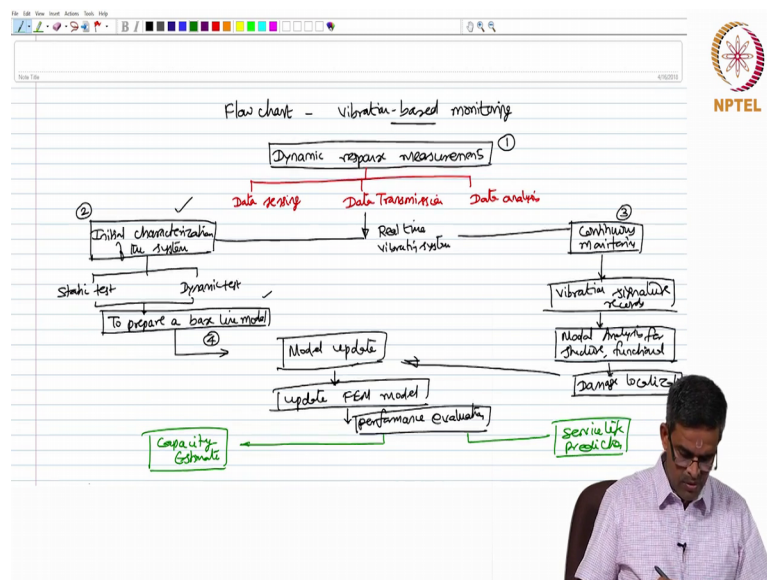


Structural Health Monitoring (SHM)
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Lecture - 27
Structural Health monitoring methods:1- Part 1

Friends, welcome to the 6th lecture in module 2, where we will talk about Structural Health Monitoring methods. There are many methods, which we will discuss in subsequent lectures. So we call this as SHM methods, lecture number one.

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Let us look into the flowchart, which is useful for vibration based monitoring. Let us say in the first step we talk about dynamic response measurements that is the first stage what we do which will be generally done through and transfer through data sensing. Then the sensed data is transmitted, and the data is analyzed. Based on this, we will be able to reach and apply it to real time vibrating system. So, we will be able to get initial characterization of the system; this is the second step. Parallely, one can also do continuous monitoring. So, in the initial characterization, we can achieve the results by two ways; one is the static test, other is the dynamic test. Both of these data will be useful to prepare a baseline model.

Similarly, from the continuous monitoring, one can achieve the vibration signature records, which can be useful in doing modal analysis for structure, which is functional

that is operational. This modal analysis for the structure under operation will be different from the conventional characterization of the system based on which the baseline modal has been prepared. So, based on this value, one can then achieve damage localization. Both these data put together, will be update the model.

So, based on the updated model, we will further update the finite element model for analysis, and based on which the performance evaluation of the system is done. The performance evaluation will finally lead towards two issues, a capacity building or capacity estimate of the structural system, and service life prediction of the system. Friends, this shows a clear flow chart, which is generally practiced for vibration based monitoring.

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I. Method using frequency & mode shapes

- change in structural characteristics can be readily identified by noticing the change in natural frequencies
- change in $[k]$, $[m]$, changes eigenvalues, which can be modelled as:

$$\{z\} = [F]\{\alpha\} + [G]\{\beta\} \quad (1)$$

where $\{z\}$ = vector of measured frequency changes
 $\{\alpha\}, \{\beta\}$ = vectors of the changes in the system $\left\{ \begin{array}{l} \text{stiffness} \\ \text{mass} \end{array} \right.$ respectively
 $[F], [G]$ = sensitivity matrices

To calculate the changes in $[k]$ & $[m]$ is to compute $\{\alpha\}, \{\beta\}$ one need to calculate the sensitivity matrices.

Now, let us see different methods, which are used for structural health monitoring for estimating the baseline model, and initial characterization. The first one is method using frequencies and mode shapes. It is vital to understand that, change in structural characteristics can be readily identified by noticing the change in natural frequencies. For example, change in stiffness, mass, changes eigenvalues, which can be modelled as where the Z vector is the vector of measured frequency changes. Alpha, and beta are vectors, indic of the changes in the system related to stiffness, and mass respectively. F and G are called sensitivity matrices.

Now, to calculate the changes in stiffness and mass, that is to compute the alpha, and beta vector; one need to calculate the sensitivity matrices.

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Either be calculated theoretically from the eigenvalues of the system
 OR
 they can be computed numerically using 'perturbation method' with
 finite element model

- studies conducted earlier show that change in Mass matrix, before and after damage is negligible
- Therefore, sensitivity equation, can be reformulated as below:

$$Z_i = \sum_{j=1}^{NE} F_{ij} \alpha_j \quad \text{--- (2)}$$

So, they can be either calculated, theoretically from the eigenvalues of the structural system or they can be computed numerically using with finite element model. However, research studies conducted earlier show that change in mass matrix, before and after damage is negligible. Therefore, the sensitivity equation can be reformulated as below.

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where Z_i - fractional change in i^{th} eigenvalue (frequency)
 α_j - fractional reduction in j^{th} stiffness parameter
 F_{ij} is expressed as fraction of Nodal Strain Energy in i^{th} mass, and j^{th} element stiffness

F_{ij} can be expressed as below:

$$F_{ij} = \frac{\{\phi_i\}^T [k_j] \{\phi_i\}}{[\phi_i]^T [k] \{\phi_i\}} \quad \text{--- (3)}$$

where $[K]$ & $[k_j]$ are global & element stiffness matrices, respectively

Where Z_i indicates the fractional change of i th eigenvalue that is, frequency. α_j indicates the fractional reduction in j -th stiffness parameter. And F_{ij} is expressed as fraction of modal strain energy for the i -th mode, stored in j -th element of the structure. F_{ij} can be expressed as below. $F_{ij} = \frac{\phi_i^T K_j \phi_i}{\phi_i^T K \phi_i}$, where K and K_j are global and elemental stiffness matrix, respectively.

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Once, stiffness matrix of the complete system and mode shape are known, then F_{ij} , as seen from Eq(3) can be generated numerically.

To obtain the relative damage, sensitivity Eqn can be reformulated as:

$$\frac{Z_m}{Z_n} = \frac{\sum_{j=1}^{NE} F_{mj} \alpha_j}{\sum_{j=1}^{NE} F_{nj} \alpha_j} \quad \text{--- (4)}$$

Suppose, only 1 element is damaged, then the above Eqn reduces to

$$\frac{Z_m}{Z_n} = \frac{F_{mq}}{F_{nq}} \quad \text{--- (5)}$$

which is unique for the q 'th location. Damage can be now located.

Once, stiffness matrix of the complete structural system, and mode shape are known, then F_{ij} , as seen from equation 3 can be generated numerically. Once you determine this, to obtain further the relative damage, sensitivity equation can be reformulated as given below; Z_m / Z_n is given by $\sum_{j=1}^{NE} F_{mj} \alpha_j / \sum_{j=1}^{NE} F_{nj} \alpha_j$. Suppose, only one element is damaged, then the above equation reduces to Z_m / Z_n is equal to F_{mq} / F_{nq} , which is unique for the q -th location so that is how the damage can be now located.

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Error index is given by:

$$e_{ij} = \frac{Z_m}{\sum_{k=1}^{NM} Z_k} - \frac{F_{mj}}{\sum_{k=1}^{NM} F_{kj}} \quad (6)$$

$e_{ij} = 0$, which indicates damage at the j 's location.

Damage sensitivity is given by:

$$\frac{\Delta \omega_i^2}{\omega_i^2} = \eta S_{ij} \left(\frac{a_k}{H} \right)_i \quad (7)$$

In working on the whole equation, the error index is given by, e_{ij} is 0, which indicates damage at the j -th location.

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where $\frac{\Delta \omega_i^2}{\omega_i^2}$ is fractional change in eigenvalue in the i 's mode

$\left(\frac{a_k}{H} \right)_i$ is dimensionless crack size, which is normalized to the depth of the member (H).

η - shape factor, accounts for geometry of the mass

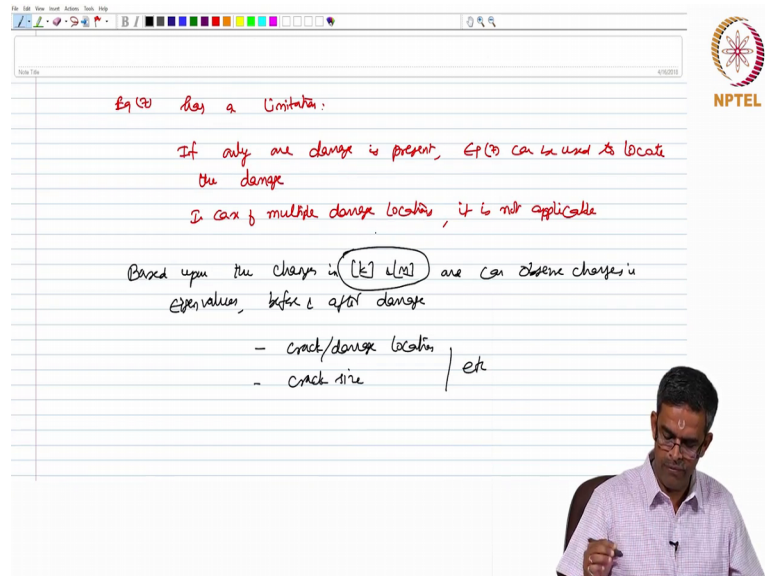
S_{ij} is sensitivity of k 's location in the i 's modal strain energy

If there is a fractional change in eigenvalue of the system, which is measured experimentally, then one can easily determine the crack size from eq (7).

Further, damage sensitivity is given by where $\Delta \omega_i^2 / \omega_i^2$ is a fractional change in eigenvalue; a_k / H is dimensionless crack size, which is normalized to the depth of the member, which is in this case H . η is the shape factor, accounting for geometry of the mass. S_{ij} is sensitivity of k -th location in the j -th modal strain energy. If there is a

fractional change in eigenvalue of the system, which is measured experimentally, then one can easily determine the crack size from equation 7, equation 7 is this value.

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Eq (7) has a Limitation:

If only one damage is present, Eq(7) can be used to locate the damage

In case of multiple damage locations, it is not applicable

Based upon the changes in $[k]$ & $[m]$ one can observe changes in eigenvalues, before & after damage

- crack/damage location
- crack size

etc

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But, this equation has a limitation. If only one damage is present, equation 7 can be used to locate the damage. In case of multiple damage locations, it is not applicable, so that is one of the limitations of equation 7.

We have now seen friends that based upon, based upon the changes in stiffness and mass matrix; one can observe changes in eigenvalues before and after damage. Then one can identify the crack or let us say the damage location, crack size etcetera as we explained in the previous slides. Now, the interesting point comes, how to find this changed k and m for different models.