Structural Health Monitoring (SHM)
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Lecture – 29
Damage identification using lumped mass and
Element modal stiffness – Part 1

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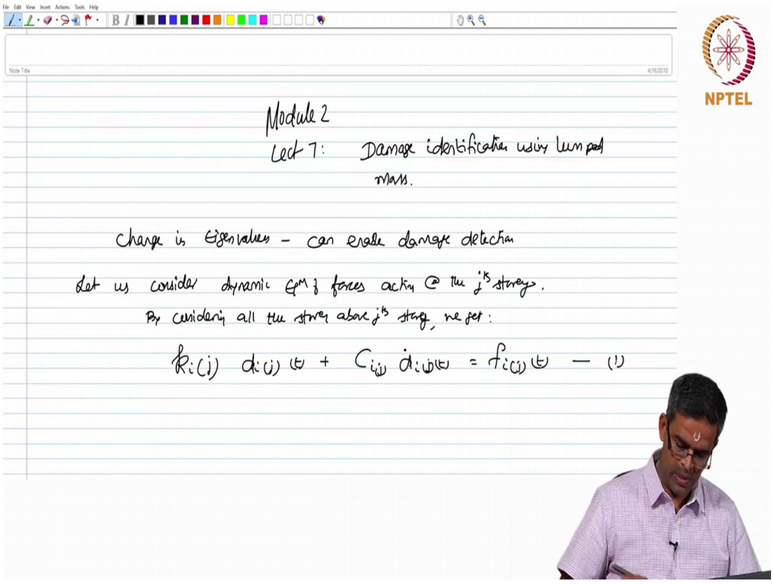

Friends, welcome to the lecture in Module 2, lecture 7. In this lecture, we will discuss about Damage identification using lumped mass modal. We already saw in the last lecture, change in Eigen values can enable damage deduction.

Let us now consider the dynamic equilibrium of forces acting at the j th storey, by considering all the storey above j th storey, we get k i j d i j of t plus c i j d dot i j of t is equal to f i j of t and say equation 1.

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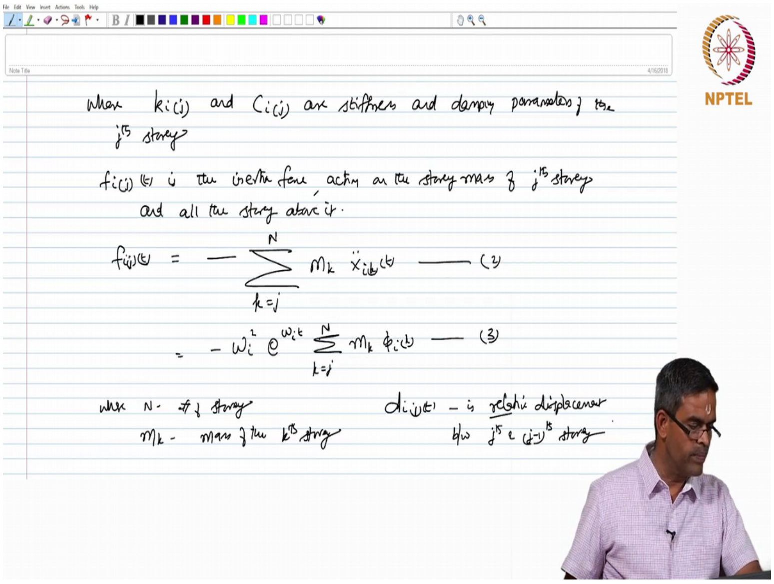

Where, k i j and c i j are stiffness and damping parameters of the j th storey; f i j of t is the inertia force acting on the story mass of j th storey and all the storey above it; f i j of t is given by k equals j to N, M k x double dot i k of t which is equal to minus omega i square e omega i t summation k equals j to N of M k phi i k.

Where N is the number of storey, M k is mass of the k th storey d i j of t is the relative displacement; that is very important; the relative displacement between j th and j minus 1 th storey.

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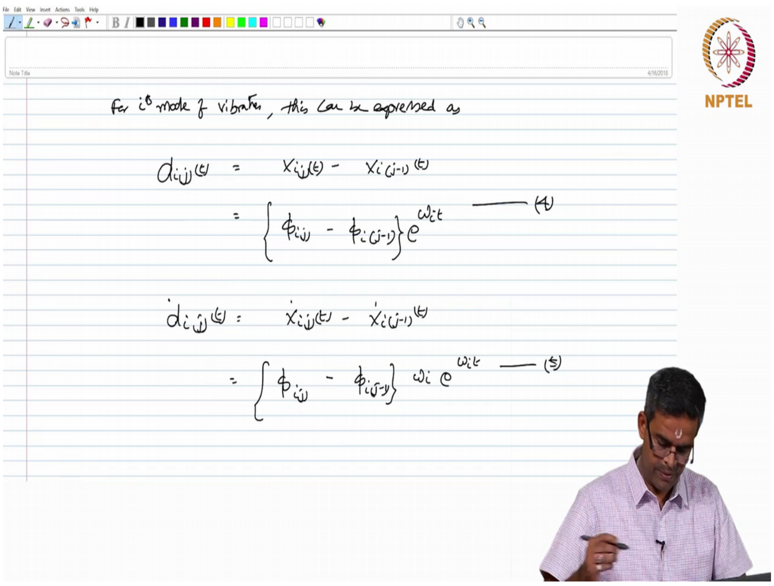

For i th mode of it is vibration, this can be expressed as d i j of t is x i j of t minus x i j minus 1 of t which will be phi i j minus phi i j minus 1, e lambda i t; d dot i j of t is given by x dot i j t minus x dot i j minus 1 t, which can be given by phi i j minus phi i j minus 1 of omega i e omega i t. In fact, this should be also omega i t.

Let us call this equation as 4 and 5.

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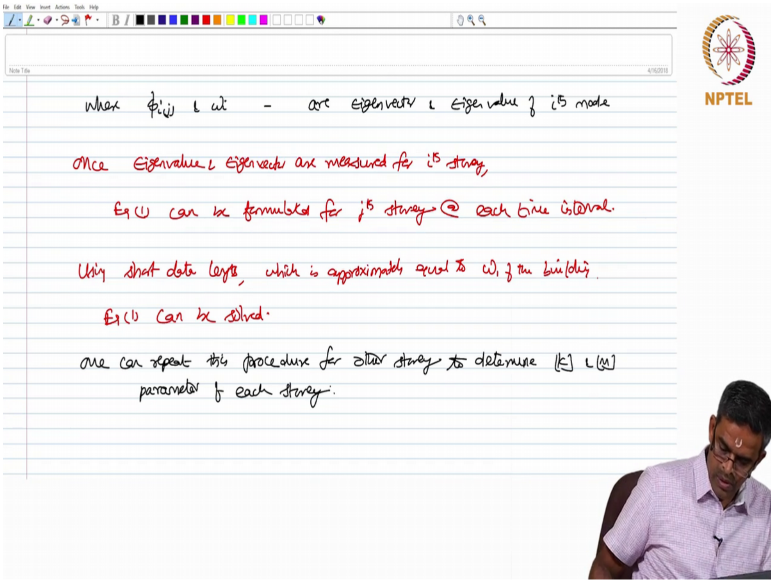

Where, phi i j and omega i are Eigenvector and Eigenvalue of i th mode. Once, the Eigenvector and Eigenvalue are measured for i th storey, equation 1 can be formulated for j th storey at each time interval using short data length which is approximately equal to natural frequency of the building; equation 1 can be solved.

One can repeat this procedure for other stories to determine stiffness and mass parameter of each storey.

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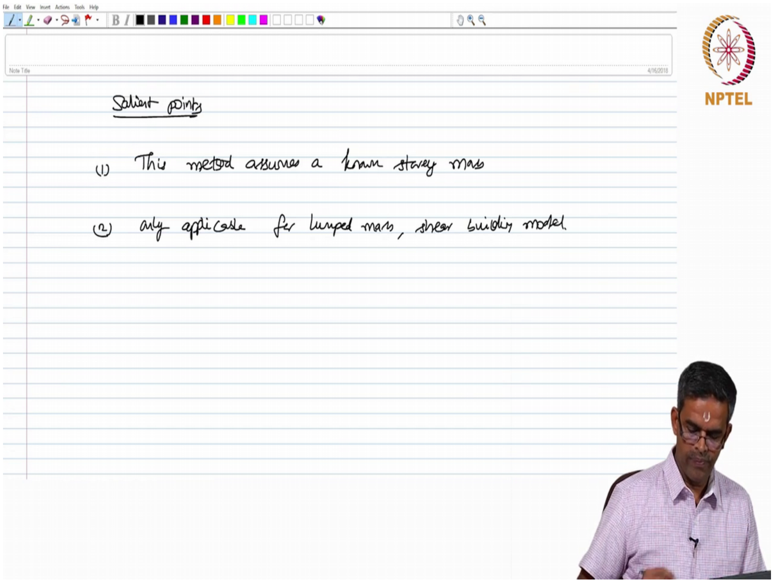

There are some salient points of this method. The first would be, this method assumes a known story mass. This method is applicable for lumped mass, shear building model. Let us see the third method, method to identify damage using element modal stiffness.

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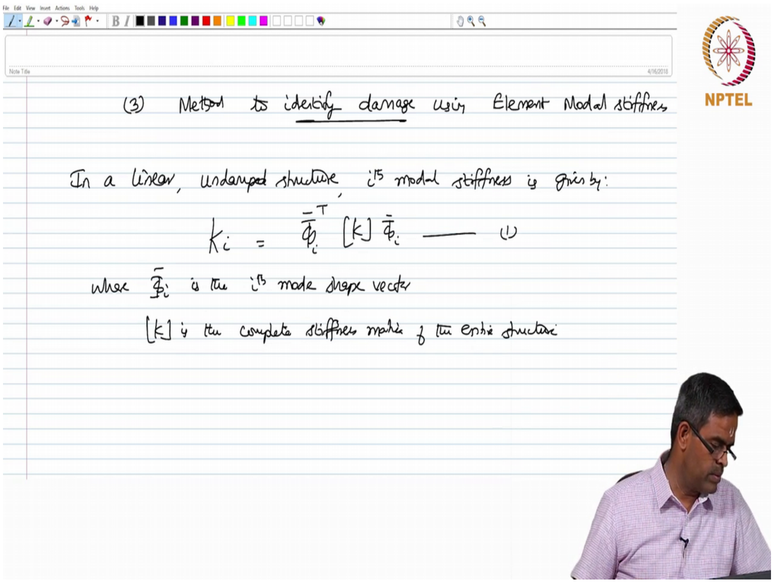

In the linear undamped structure, i th modal stiffness is given by k i is phi i transpose phi i bar transpose k phi; where, phi i bar is the i th mode shape vector, k is the complete stiffness matrix of the entire structure.

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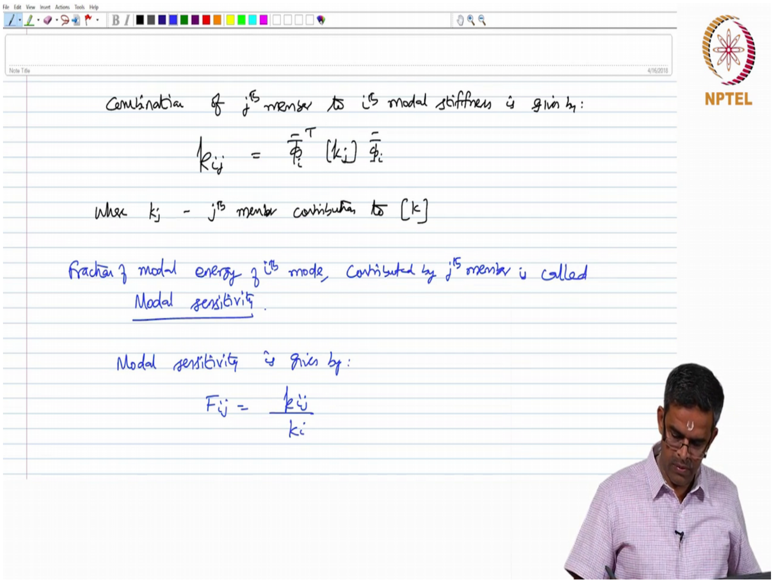

Now, combination of j th member to i th modal stiffness is given by k i j is phi bar i transpose k j of phi i bar where, k j is the j th member stiffness, j th member contribution to the total K.

Now, fraction of modal energy of i th mode contributed by j th member is called Modal sensitivity. Modal sensitivity is given by F i j is k i j by k i.

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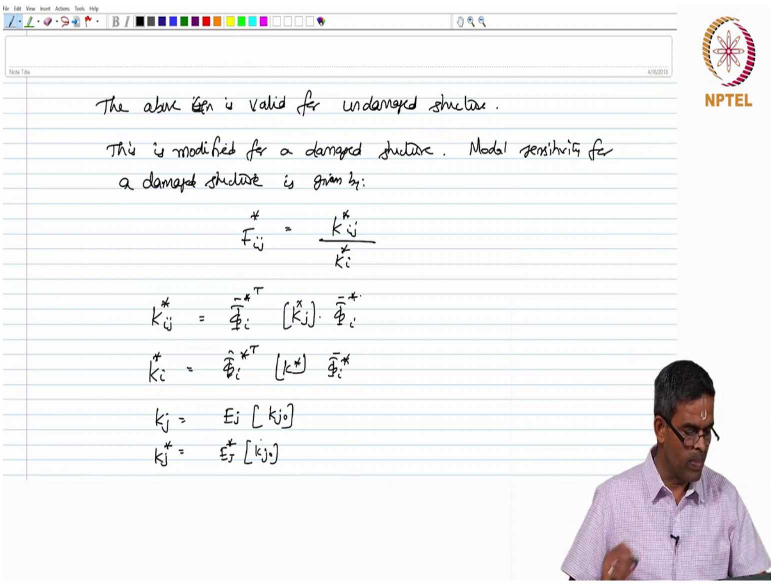

The above equation is valid for undamaged structure. This is modified for a damaged structure. The modal sensitivity for a damaged structure is given by, F i j star is k i j star by k i star; k i j star is phi bar star transpose k star j phi bar star and k i star is phi bar i transpose k star phi bar i star; k j is therefore, expressed as E j of k j 0 and k j star is E j star of k j 0.

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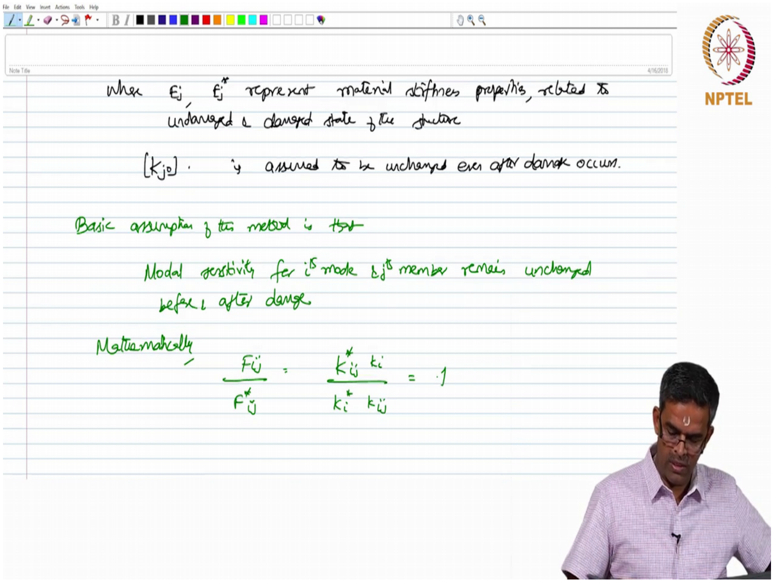

Where E j and E j star represent, material stiffness property related to undamaged and damaged state of the structure; k j 0 is assumed to be unchanged even after damage occurs. This method has a basic assumption. The basic assumption of this method is that, Model sensitivity for i th mode and j th member remain unchanged before and after damage.

Mathematically, F i j by F star i j, that is damaged and undamaged state is k i j star k i by k i star k i j which is unity.

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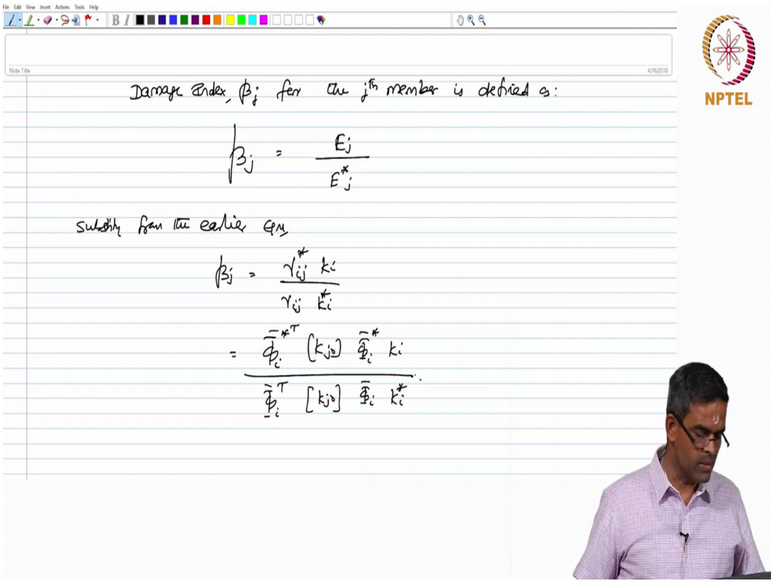

Now, the damage index, beta j for the j th member is defined as beta j is E j by E star j. Substituting, from the earlier equations, beta j can be expressed as, i j star k i by i j k i star, which is phi bar i star transpose k j 0 phi bar star k i divided by phi bar i transpose k j phi bar i k i star.

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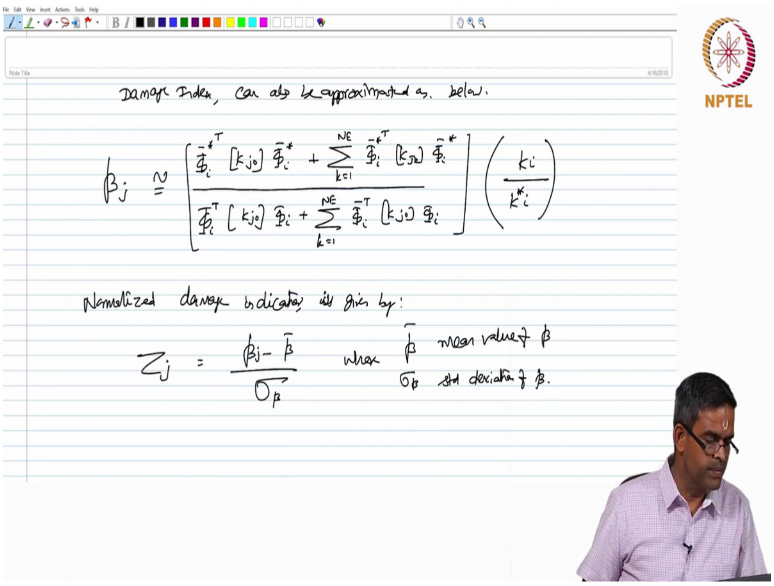

Now, further, the damage index can also be approximated as below; beta j is approximately equal to phi bar i star transpose k j naught phi bar i star plus summation of k equals 1 to N E phi bar transpose k j 0 phi bar star divided by phi i transpose k j 0 phi i plus k equals 1 to N E phi i transpose k j 0 phi i multiplied by k i by k star i.

One can now obtain the normalized damage indicator is given by z j which is beta j minus beta bar by sigma beta where beta bar is the mean value of the damage index.

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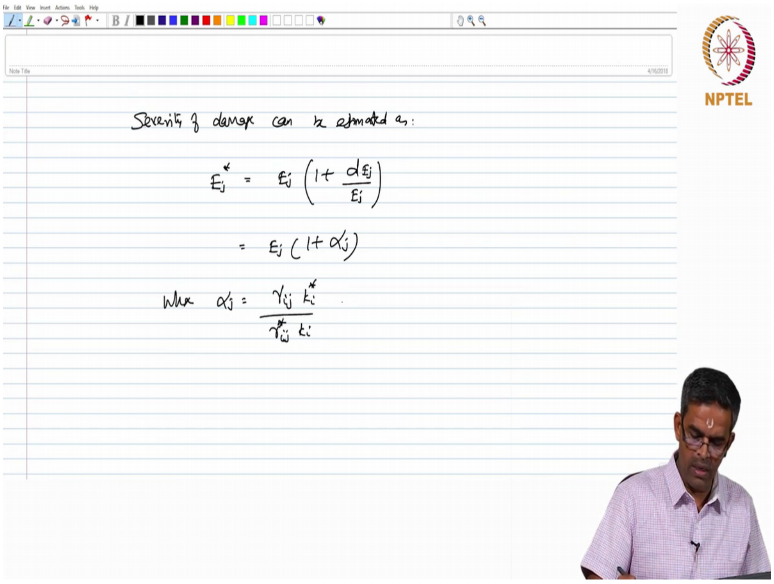

And sigma beta is the standard deviation of beta. Severity of damage can be estimated as E j star is E j 1 plus d E j by E j which will be E j 1 plus alpha j where alpha j is gamma i j k i star by gamma i j star k i minus 1.